A new 6-DOF parallel robot with simple kinematic model

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***Abstract*—This paper presents a novel six-legged parallel robot, the kinematic model of which is simpler than that of the simplest six-axis serial robot. The new robot is the 6-DOF extension of the Cartesian parallel robot. It consists of three pairs of base-mounted prismatic actuators, the directions in each pair parallel to one of the axes of a Cartesian coordinate system. In each of the six legs, there are also two passive revolute joints, the axes of which are parallel to the direction of the prismatic joint. Finally, each leg is attached to the mobile platform via a spherical joint. The direct kinematics of the novel parallel robot can be solved easily by partitioning the orientation and the position of the mobile platform. There are eight distinct solutions, which can be found directly by solving a linear system and alternating the signs of three radicals. This parallel robot has a large workspace and is suitable for machining or rapid prototyping, as detailed in this paper.**

***Keywords:* Cartesian parallel robot, direct kinematics, singularities, rapid prototyping.**

I. Introduction

The invention of the Cartesian parallel robot [1], also known as the Tripteron (Fig. 1), in 2001 (see [2] for more details), was a key turning point in the theory of parallel mechanisms. Different designs of parallel robots had been known for decades, but none as simple as the Cartesian parallel robot, the kinematic model of which resumes to

$$x = \rho_1,\ y = \rho_2,\ z = \rho_3, \quad (1)$$

where ρ_1, ρ_2, and ρ_3 are the active-joint variables, and x, y, and z are the position coordinates of the mobile platform. Later, the concept was extended to 4 DOFs (degrees of freedom) with the four-legged Quadrupteron robot [4], but the kinematic model of the latter is no more linear. Its direct kinematics problem requires the solution of a quadratic equation. Later, the Tripteron concept was extended to 5 DOFs with the five-legged Pentapteron [5,6]. Its forward kinematics, however, boils down to solving a polynomial of degree 1680, and allows at least 208 real solutions [7]. Extending the concept of the simplest parallel robot to 5 DOFs yielded one of the most complex parallel robots.

We present in this article the 6-DOF missing member of the so-called Multipteron family. Contrary to expectations, the kinematic model of the Hexapteron, as we call it, is not complex. In fact, it is simpler than the kinematic model of the simplest 6-DOF serial robot. Furthermore, the Hexapteron can have a fairly large workspace, if properly designed.

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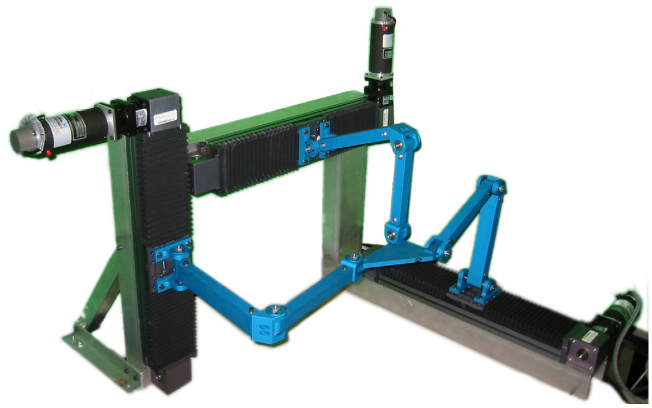

Figure 1. The Tripteron Cartesian parallel robot [2].

In the next section, we present the Hexapteron's geometric design and solve its inverse and direct kinematics. Then, in Section III, we find its singularity loci and discuss its workspace. Section IV presents the mechanical design of a 3D printing device based on the Hexapteron architecture. Finally, we present our conclusions in Section V.

II. Kinematic Modeling of the Hexapteron

The Hexapteron consists of six legs, each composed of a cylindrical joint, a revolute joint, and a spherical joint, arranged orthogonally as shown in Fig. 2, where *O-xyz* is the base reference frame and *C-x′y′z′* is the mobile reference frame. In each leg, the axes of the two passive revolute joints are parallel to the direction of the actuated prismatic joint.

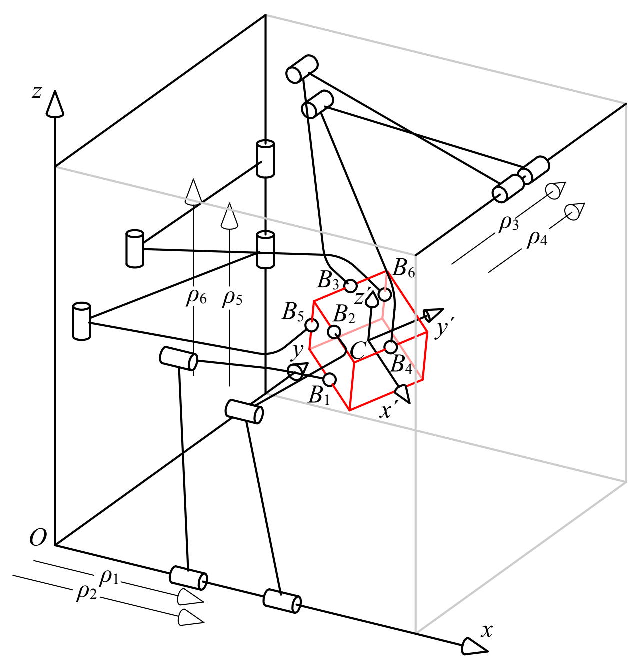

Figure 2. Schematic of the Hexapteron.

The coordinates of the centers of the spherical joints, *i.e.*, of points B_i (i = 1, 2, …, 6), expressed in the mobile frame, are given with the following vectors:

$$\mathbf{r}_{B_1} = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, \ \mathbf{r}_{B_2} = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \ \mathbf{r}_{B_3} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \ \mathbf{r}_{B_4} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{r}_{B_5} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}, \ \mathbf{r}_{B_6} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}. \tag{2}$$

There are other arrangements of the platform joints that lead to mechanisms with kinematic models of similar simplicity, but this arrangement seems to lead to larger workspace. Also, note that at zero orientation (when the orientations of the mobile frame and the base frame are the same), each pair of adjacent legs would interfere, unless they are designed with an offset or curved, as illustrated in Fig. 2. This offset, however, does not change the kinematics of the mechanism, since each active joint variable ρ_i will be defined as the distance between B_i and the plane passing through O and normal to the direction of actuator i.

Thus, the kinematic model of the Hexapteron is represented by the following six simple equations:

$$\mathbf{i}^T \left(\mathbf{p} + \mathbf{R}\mathbf{r}_{B_1}\right) = \rho_1, \tag{3}$$

$$\mathbf{i}^T \left(\mathbf{p} + \mathbf{R}\mathbf{r}_{B_2}\right) = \rho_2, \tag{4}$$

$$\mathbf{j}^T \left(\mathbf{p} + \mathbf{R}\mathbf{r}_{B_3}\right) = \rho_3, \tag{5}$$

$$\mathbf{j}^T \left(\mathbf{p} + \mathbf{R}\mathbf{r}_{B_4}\right) = \rho_4, \tag{6}$$

$$\mathbf{k}^T \left(\mathbf{p} + \mathbf{R}\mathbf{r}_{B_5}\right) = \rho_5, \tag{7}$$

$$\mathbf{k}^T \left(\mathbf{p} + \mathbf{R}\mathbf{r}_{B_6}\right) = \rho_6, \tag{8}$$

where $\mathbf{p} = [x, y, z]^T$ is the position vector of point C with respect to the base frame, $\mathbf{R}$ is the 3×3 orthogonal rotation matrix representing the orientation of the mobile frame with respect to the base frame, and **i**, **j**, and **k** are the unit vectors along, respectively, the x, y, and z axis of the base frame.

Equations (3–8) represent the solution of the inverse kinematics of the Hexapteron. Obviously, there is only one solution per leg, and serial singularities occur whenever a leg is fully stretched or fully folded.

In order to solve the direct kinematics, we need to manipulate (3–8), but first we need to choose a particular orientation representation. As usual, some orientation representations make the direct kinematics more difficult to solve, while others lead to elegant solutions. For example, among all Euler angles conventions, only the ZYX fixed angle set leads to relatively compact direct kinematic equations. However, we will use Euler parameters, since the resulting direct kinematic equations are even more compact and symmetric.

Based on Euler parameters, the rotation matrix is expressed as follows:

$$\mathbf{R} = \begin{bmatrix} 1-2q_2^2-2q_3^2 & 2(q_1q_2-q_0q_3) & 2(q_1q_3+q_0q_2) \\ 2(q_1q_2+q_0q_3) & 1-2q_1^2-2q_3^2 & 2(q_2q_3-q_0q_1) \\ 2(q_1q_3-q_0q_2) & 2(q_2q_3+q_0q_1) & 1-2q_1^2-2q_2^2 \end{bmatrix}, \tag{9}$$

where the four Euler parameters satisfy the following constraint equation

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1. \tag{10}$$

Next, we will get rid of the position coordinates by simply subtracting (3) from (4), (5) from (6), and (7) from (8), and then dividing each new equation by 2, which yields:

$$2q_1q_3 + 2q_0q_2 = \rho_{21}, \tag{11}$$

$$2q_1q_2 + 2q_0q_3 = \rho_{43}, \tag{12}$$

$$2q_2q_3 + 2q_0q_1 = \rho_{65}, \tag{13}$$

where

$$\rho_{21} = (\rho_2 - \rho_1)/2, \ \rho_{43} = (\rho_4 - \rho_3)/2, \ \rho_{65} = (\rho_6 - \rho_5)/2.$$

There are different avenues for solving (10–13), including the use of Gröbner basis, but the following approach is by far the most elegant one. We obtain four new equations by

- subtracting (12) and (13) from the sum of (10) and (11);
- subtracting (11) and (13) from the sum of (10) and (12);
- subtracting (11) and (12) from the sum of (10) and (13);
- adding (10), (11), (12) and (13).

Factoring the left sides of these four new equations yields:

$$(q_0 - q_1 + q_2 - q_3)^2 = 1 + \rho_{21} - \rho_{43} - \rho_{65}, \tag{14}$$

$$(q_0 - q_1 - q_2 + q_3)^2 = 1 - \rho_{21} + \rho_{43} - \rho_{65}, \tag{15}$$

$$(q_0 + q_1 - q_2 - q_3)^2 = 1 - \rho_{21} - \rho_{43} + \rho_{65}, \tag{16}$$

$$(q_0 + q_1 + q_2 + q_3)^2 = 1 + \rho_{21} + \rho_{43} + \rho_{65}. \tag{17}$$

Of course, the direct kinematics of the Hexapteron will have a solution only if the right hand side of each of the above four equations is nonnegative.

In order to solve (14–17), we take the square roots on each side, which yields the following system of four linear equations in the Euler parameters:

$$Q_1 \equiv q_0 - q_1 + q_2 - q_3 = \delta_1\sqrt{1 + \rho_{21} - \rho_{43} - \rho_{65}} \equiv \delta_1\sqrt{\Gamma_1}, \tag{18}$$

$$Q_2 \equiv q_0 - q_1 - q_2 + q_3 = \delta_2\sqrt{1 - \rho_{21} + \rho_{43} - \rho_{65}} \equiv \delta_2\sqrt{\Gamma_2}, \tag{19}$$

$$Q_3 \equiv q_0 + q_1 - q_2 - q_3 = \delta_3\sqrt{1 - \rho_{21} - \rho_{43} + \rho_{65}} \equiv \delta_3\sqrt{\Gamma_3}, \tag{20}$$

$$Q_4 \equiv q_0 + q_1 + q_2 + q_3 = \delta_4\sqrt{1 + \rho_{21} + \rho_{43} + \rho_{65}} \equiv \delta_4\sqrt{\Gamma_4}, \tag{21}$$

where $\delta_1 = \pm 1$, $\delta_2 = \pm 1$, $\delta_3 = \pm 1$, $\delta_4 = \pm 1$, and $\Gamma_1 = 1 + \rho_{21} - \rho_{43} - \rho_{65}$, $\Gamma_2 = 1 - \rho_{21} + \rho_{43} - \rho_{65}$, etc.

Finally, solving the set of four linear equations (18–21) yields

$$q_0 = \frac{1}{4}\left(\delta_1\sqrt{\Gamma_1} + \delta_2\sqrt{\Gamma_2} + \delta_3\sqrt{\Gamma_2} + \delta_4\sqrt{\Gamma_4}\right), \quad (22)$$

$$q_1 = \frac{1}{4}\left(-\delta_1\sqrt{\Gamma_1} - \delta_2\sqrt{\Gamma_2} + \delta_3\sqrt{\Gamma_2} + \delta_4\sqrt{\Gamma_4}\right), \quad (23)$$

$$q_2 = \frac{1}{4}\left(\delta_1\sqrt{\Gamma_1} - \delta_2\sqrt{\Gamma_2} - \delta_3\sqrt{\Gamma_2} + \delta_4\sqrt{\Gamma_4}\right), \quad (24)$$

$$q_3 = \frac{1}{4}\left(-\delta_1\sqrt{\Gamma_1} + \delta_2\sqrt{\Gamma_2} - \delta_3\sqrt{\Gamma_2} + \delta_4\sqrt{\Gamma_4}\right). \quad (25)$$

However, since the quaternion $[q_0, q_1, q_2, q_3]$ is equivalent to the quaternion $[-q_0, -q_1, -q_2, -q_3]$, we can eliminate δ_4 (*i.e.*, set it equal to 1), which means that we have only eight distinct solutions for the orientation of the mobile platform.

Now that we know the orientation of the mobile platform, we can obtain its position by adding (3) to (4), (5) to (6) and (7) to (8) and rearranging:

$$x = 2q_1q_2 - 2q_0q_3 + \frac{1}{2}(\rho_1 + \rho_2), \quad (26)$$

$$y = 2q_0q_1 - 2q_2q_3 + \frac{1}{2}(\rho_3 + \rho_4), \quad (27)$$

$$z = 2q_1q_3 - 2q_0q_2 + \frac{1}{2}(\rho_5 + \rho_6). \quad (28)$$

Fig. 3 shows a top view of the eight poses of the mobile platform for a numerical example where $\rho_1 = 4.7$, $\rho_2 = 5.3$, $\rho_3 = 5.4$, $\rho_4 = 4.6$, $\rho_5 = 4.8$, and $\rho_6 = 5.2$. For clarity, the legs are not shown (though the first four legs are hinted by dashed lines), but one can easily imagine that not all poses are feasible on a real prototype. Note that Fig. 2 shows the first solution, which is feasible. In addition to Fig. 3, Table 1 presents the corresponding position coordinates and Euler angles, according to the ZYX fixed angle set, where

$$\mathbf{R}(\psi, \theta, \phi) = \mathbf{R}_x(\phi)\mathbf{R}_y(\theta)\mathbf{R}_z(\psi). \quad (29)$$

TABLE I. AN EXAMPLE OF THE EIGHT SOLUTIONS TO THE DIRECT KINEMATIC PROBLEM OF THE HEXAPTERON

Solution #	**Pose of the mobile frame**					
	x	y	z	ψ	θ	ϕ
1	5.53	5.40	4.54	−33.92°	17.46°	25.07°
2	5.86	5.79	4.18	−64.93°	17.46°	56.08°
3	4.13	5.79	5.82	64.93°	17.46°	123.92°
4	4.47	5.40	5.46	33.92°	17.46°	154.93°
5	4.47	4.60	4.54	−33.92°	162.54°	25.07°
6	4.14	4.21	4.18	−64.93°	162.54°	56.08°
7	5.86	4.21	5.82	~~64.93°~~	~~162.54°~~	~~123.92°~~
8	5.53	4.60	5.46	33.92°	162.54°	154.93°

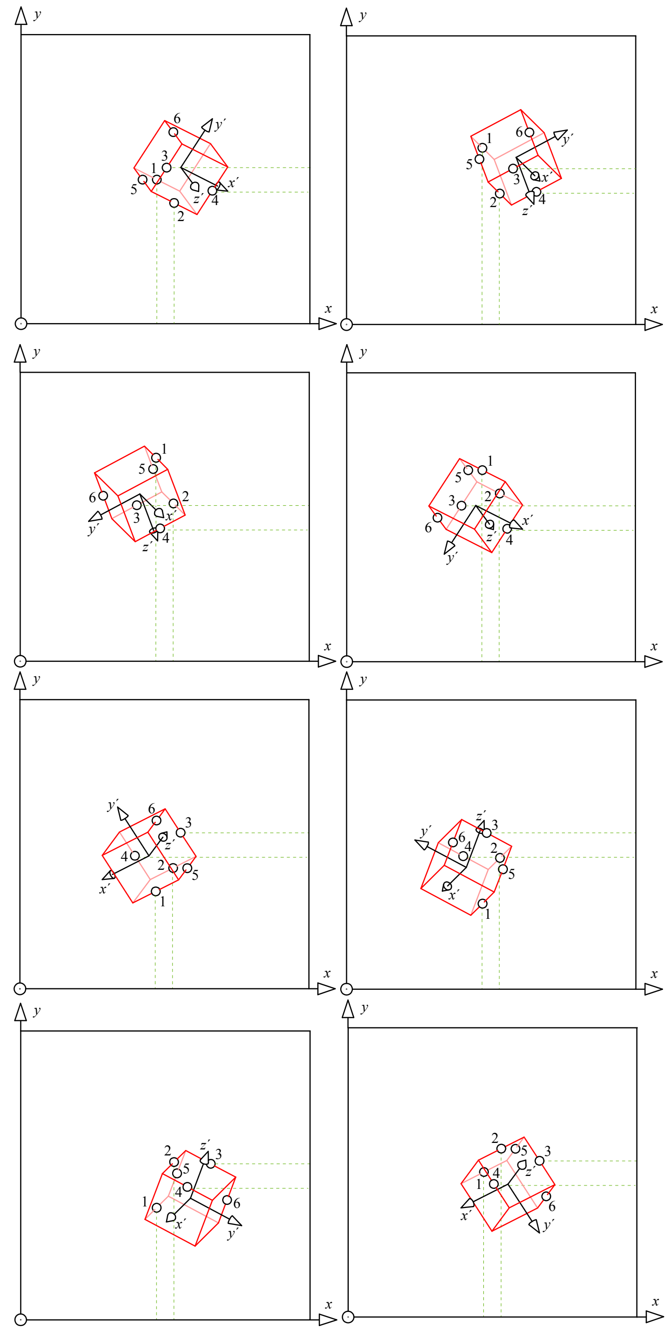

Figure 3. Top view of the eight solutions to the direct kinematic problem presented in Table 1 (legs not shown).

It is important to note that the direct kinematics of the Hexapteron is a special case of the six-points-on-six-planes problem studied in [8] and, to a certain extent, in [9]. Indeed, the author of [8] essentially proved that the direct kinematics of a Hexapteron in which the spatial arrangement of the six platform joints and the directions of the actuated prismatic joints are arbitrary is equivalent to solving a univariate polynomial of degree 8. Thus, what is particularly interesting about our Hexapteron is that each of its eight direct kinematic solutions can be obtained directly, without solving any polynomial. What is even more important is that we can select which assembly mode we are interested in and calculate the corresponding unique direct kinematic solution, as done in [10] for the Agile Eye, by specifying δ_1, δ_2, and δ_3.

Of course, the Hexapteron is not the only 6-DOF parallel robot with trivial direct kinematics. However, virtually all other such parallel mechanisms are either three legged (*e.g.*, [11–13]) or asymmetrical (*e.g.*, [14–15]) or require double or triple spherical joints (*e.g.*, [15]).

III. Singularities and Workspace of the Hexapteron

As already mentioned, the Hexapteron is at a serial singularity whenever one of its six legs is fully stretched or fully folded (*i.e.*, when in a leg, the center of the spherical joint is coplanar with the axes of the two passive revolute joints). In order to find the parallel singularities, we should obtain the Jacobian matrix and examine the conditions that lead to zeroing of its determinant.

Skipping the derivations, the Jacobian matrix of the Hexapteron is

$$\mathbf{J} = \begin{bmatrix} (\mathbf{R}\mathbf{r}_{B_1})\times\mathbf{i} & \mathbf{i}^T \\ (\mathbf{R}\mathbf{r}_{B_2})\times\mathbf{i} & \mathbf{i}^T \\ (\mathbf{R}\mathbf{r}_{B_3})\times\mathbf{j} & \mathbf{j}^T \\ (\mathbf{R}\mathbf{r}_{B_4})\times\mathbf{j} & \mathbf{j}^T \\ (\mathbf{R}\mathbf{r}_{B_5})\times\mathbf{k} & \mathbf{k}^T \\ (\mathbf{R}\mathbf{r}_{B_6})\times\mathbf{k} & \mathbf{k}^T \end{bmatrix}. \quad (30)$$

The determinant of this matrix is

$$\det(\mathbf{J}) = -8Q_1Q_2Q_3Q_4 = \pm\sqrt{\Gamma_1}\sqrt{\Gamma_2}\sqrt{\Gamma_3}\sqrt{\Gamma_4}, \quad (31)$$

which means that the Hexapteron is at a parallel singularity whenever $Q_j = 0$, *i.e.*, whenever $\Gamma_j = 0$ ($j = 1, 2, 3, 4$).

Fig. 4 shows that the four planes $\Gamma_j = 0$ form a pyramid in the ρ_{21}, ρ_{43}, ρ_{65} active-joint space. Note that the interior of that pyramid is the allowable space, where $\Gamma_j > 0$ ($j = 1, 2, 3, 4$), *i.e.*, where the Hexapteron has eight assembly modes.

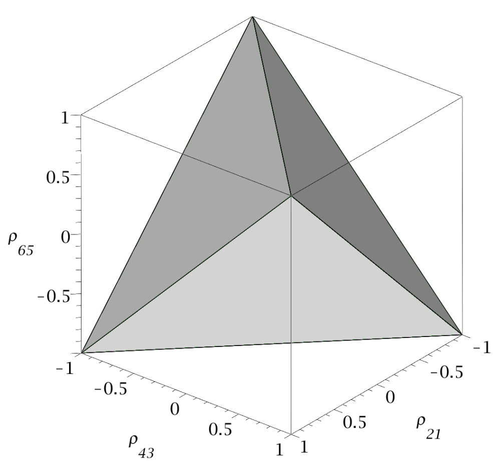

Figure 4. Singularity surfaces of the Hexapteron in its active-joint space.

Finally, note that if we use the ZYX fixed angle set, then the determinant of the Jacobian matrix is

$$\det(\mathbf{J}) = -8\cos(\theta)^2\cos(\phi+\psi)\cos(\phi-\psi). \quad (31)$$

Now, even though the Hexapteron is a 6-DOF mechanism, we are mostly interested in its 5-axis capabilities, or more specifically in the maximum tilt angles that it can achieve. It is easy to show that the angle between the z' axis of the mobile frame and the z axis and the base frame is

$$\beta = \cos^{-1}(\cos\phi\cos\theta). \quad (31)$$

Clearly, θ should always remain in the open range (−90°, 90°) to avoid parallel singularities. Similarly, it is possible to show that ϕ should also always remain in the open range (−90°, 90°) to avoid parallel singularities. Therefore, in theory, the maximum tilt angle of the mobile platform can be anything below 90°.

As for the position workspace, as shown in [2], if the legs are sufficiently long, the only theoretical limits are the strokes of the actuators. In practice, however, the main limits (particularly for the orientation workspace) are the interferences between the legs. Since these legs must have complex curved shapes in order to reduce interferences, it is very difficult to compute the workspace of the real Hexapteron.

IV. Mechanical Design of a Hexapteron for 3D Printing

Fused deposition modeling (FDM) of plastics typically limits the overhang of individual layers to less than 45°. Models requiring overhangs greater than this need printed support structures to prevent unintended part deformation. Yet, 3D printers have been shown to print upside down and in zero-gravity indicating that the 45° limit is from the normal of the layer instead of from the gravity vector.

The vast majority of 3D printers are 3-DOF robots (Cartesian serial robots or Delta parallel robots [16]) and typically print in planar layers perpendicular to gravity. The additional DOFs of the Hexapteron allow for unconventional printing of curved and tilted layers. Software will need to be written that will produce layers that intersect the surface of a model at ±45° from its normal. This will allow FDM to produce models with negative overhangs that previously required techniques such as selective laser sintering. Additionally, 3D printers have been required to print on planar surfaces. The possibility for curved and tilted layers will allow for printing directly on non-planar object. For instance, it would be possible to print a plastic handhold around the small bar on a dumbbell. Previously, this would have been impossible without disassembly of the dumbbell.

The mechanical design of the Hexapteron (Fig. 5) took advantage of the system's symmetry. Most parts are made in triple, including laser cut plates from aluminum, and the even- and odd-numbered legs, also machined from aluminum. The extremities of each leg, which provide the offsets necessary to reduce collisions, are made of steel (Fig. 6). All revolute joints are mounted on ball bearings. Finally, each of the six platform joints is made of a Cardan joint with two revolute joints on each side. This redundancy increases significantly the range of the joints as shown in Fig. 6, while avoiding a gimbal lock.

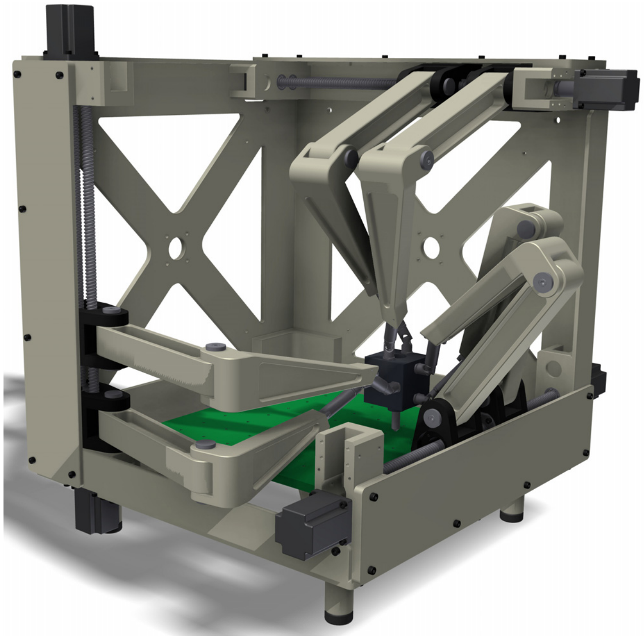

Figure 5. Mechanical design of the Hexapteron under construction.

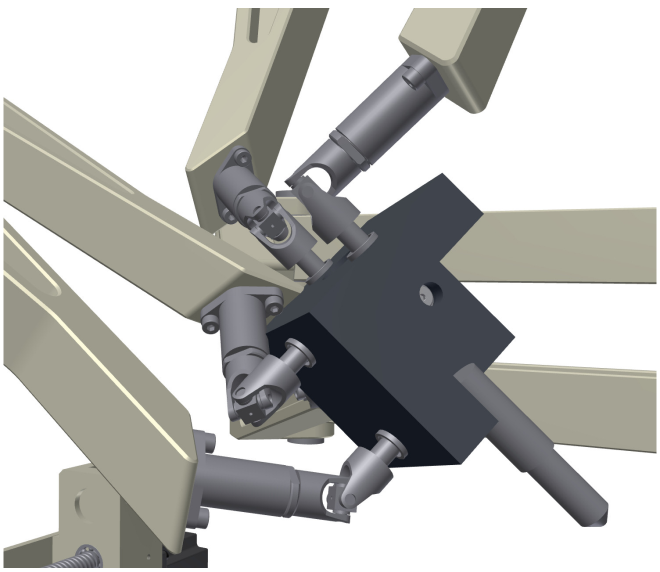

Figure 6. A closeup of the mobile platform at an extreme orientation.

For actuation, six NEMA23 step motors with maximal torque of 1.9 Nm are used. Movement to the carriages (machined from aluminum) is transmitted through three 500 mm × 20 mm linear rails and six 16 mm ball screws with 10 mm pitch. The overall dimensions of the machine are about 750 mm × 750 mm × 750 mm.

Finally, the mobile platform, also machined from aluminum, is equipped with a hot end that heats up to about 185°C as 1.75 mm filament is pressed into it. The nozzle of the hot end is of diameter 0.4 mm. Not pictured in Figs. 5 and 6 is an FEP Teflon tube (*i.e.*, a Bowden tube) that runs from the top of the mobile platform to a remote filament drive, also not pictured. Eventually, a gripper can be incorporated into the mobile platform to allow for automated assembly of printed parts and part removal.

Initial simulations show the mobile platform can tilt up to about 45° in any direction (Fig. 6). However, after analyzing the first two prototypes that are currently being machined, the mechanical design will be optimized in order to allow greater tilt angles.

V. Conclusions and Future Work

Among all parallel robots with six identical legs, the Hexapteron is probably the one with simplest kinematic model. Naturally, this first paper focused mainly on solving the direct kinematics of the Hexapteron and studying its singularities. Next, its mechanical design will be optimized to allow the largest possible tilt angle in any direction. The redundant rotational DOF about the z' should, however, be taken into account. Finally, the potential for using the Hexapteron design for building a machine tool will also be explored. Although we have not yet performed detailed workspace analyses for any particular actual prototype, there is no doubt that the Hexapteron has a larger workspace than any other hexapod system.

VI. Acknowledgements

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References

[1] X. Kong and C. M. Gosselin, "Kinematics and singularity analysis of a novel type of 3-CRR 3-DOF translational parallel manipulator," *The International Journal of Robotics Research*, Vol. 21, No. 9, September, 2002, pp. 791–798.

[2] C. M. Gosselin, X. Kong, S. Foucault, and I. A. Bonev, I.A., "A fully decoupled 3-DOF translational parallel mechanism," *2004 Parallel Kinematic Machines International Conference*, pp. 595–610, Chemnitz, Germany, April 20–21, 2004.

[3] I.A. Bonev, "The true origins of parallel robots," available at http://www.parallemic.org/Reviews/Review007.html, 2003.

[4] P.-L. Richard, C.M. Gosselin and X. Kong, "Kinematic analysis and prototyping of a partially decoupled 4-DOF 3T1R parallel manipulator," *Proceedings of the 2006 ASME Design Engineering Technical Conferences & Computers and Information in Engineering Conference*, Philadelphia, USA, September 10–13, 2006.

[5] X. Kong and C.M. Gosselin, "Type synthesis of 5-DOF parallel manipulators based on screw theory," *Journal of Robotic Systems*, Vol. 22, No. 10, 2005, pp. 535–547.

[6] C. Gosselin, M. T. Masouleh, V. Duchaine, P. L. Richard, S. Foucault, and X. Kong, "Parallel mechanisms of the Multipteron family: Kinematic architectures and benchmarking," *IEEE International Conference on Robotics and Automation*, Roma, Italy, 10–14 April, pp. 555–560 (2007).

[7] M. T. Masouleh, M. Husty, and C. Gosselin, "Forward kinematic problem of 5-PRUR parallel mechanisms using study parameters," *Advances in Robot Kinematics: Motion in Man and Machine*, Springer, pp. 211–221, 2010.

[8] C.W. Wampler, "On a rigid body subject to point-plane constraints," *Journal of Mechanical Design*, Vol. 128, No. 1, pp. 151–158, 2005.

[9] X.S. Gao, D. Lei, Q. Liao, and G.-F. Zhang, "Generalized Stewart-Gough platforms and their direct kinematics," IEEE Transactions on Robotics and Automation, Vol. 21, No. 2, pp. 141–151, 2005.

[10] X. Kong and C.M. Gosselin, "A formula that produces a unique solution to the forward displacement analysis of a quadratic spherical parallel manipulator: the Agile Eye," *Journal of Mechanisms and Robotics*, Vol. 2, No. 4, 2010.

[11] Y.K. Byun and H.-S. Cho, "Analysis of a novel 6-DOF, 3-PPSP parallel manipulator," *International Journal of Robotics Research*, Vol. 16, No. 6, pp. 859–872, 1997.

[12] C. Chen, T. Gayral, S. Caro, S. Abeywardena, D. Chablat and G. Moroz, “A six degree of freedom epicyclic-parallel manipulator,” *Journal of Mechanisms and Robotics*, Vol. 4, No. 4, 2012.
[13] SmarPods by SmarAct, http://www.smaract.de/products/smarpod, consulted February 14, 2014.
[14] C.-D. Zhang, and S.M. Song, “Forward kinematics of a class of parallel (Stewart) platforms with closed-form solutions, IEEE International Conference on Robotics and Automation, pp. 2676–2681, Sacramento, April 11–14, 1991.
[15] P. Nanua, *Direct kinematics of parallel mechanisms*, Master’s Thesis, Ohio State University, USA, 1988.
[16] I.A. Bonev, “Affordable 3D printers – Parallel robots catch on,” available at http://coro.etsmtl.ca/blog/?p=116, 2013.