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Aerodynamic Forces Approximations using the Chebyshev Method for Closed-Loop Aero-servoelasticity Studies

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Abstract

The approximation of unsteady generalized aerodynamic forces from the frequency domain into the Laplace domain acting on a Fly-By-Wire aircraft presents an important challenge in the aero-servoelasticity area. The aerodynamic forces in the reduced-frequency domain are approximated in the Laplace domain, to be able to study the effects of the control laws on the flexible aircraft structure. In this paper, we present a new method for the approximation of the generalized aerodynamic forces by use of Chebyshev polynomials and their orthogonality properties. A comparison of this new method with the Padé method used to calculate an approximation of the generalized aerodynamic forces from the frequency domain into the Laplace domain is presented. This comparison shows that this new method gives excellent results with respect to the Padé method and is applied on the Aircraft Test Model from NASA Dryden Flight Research Center.

Résumé

L'approximation de forces aérodynamiques généralisées non-stationnaires du domaine de la fréquence dans le domaine du Laplace, forces qui actionnent sur un avion a commandes électriques, représente une importante challenge pour le domaine de l'aéroservoélasticité. Les forces aérodynamiques du domaine de la fréquence réduite sont approximées dans le domaine du Laplace pour étudier les effets des lois de contrôle sur la structure flexible de l'avion. Dans cet article nous présentons une nouvelle méthode pour l'approximation de forces aérodynamiques

continued on page 2

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Nomenclature

$\mathbf{C}$	modal damping matrix
c	wing chord length
$\mathbf{K}$	modal elastic stiffness matrix
k	reduced frequency
$\mathbf{M}$	modal inertia or mass matrix
M	Mach number
T	Chebyshev polynomial
$\mathbf{Q}$	modal generalized aerodynamic forces matrix
$\mathbf{Q}_I$	imaginary part of the modal generalized aerodynamic forces matrix
$\mathbf{Q}_R$	real part of the modal generalized aerodynamic forces matrix
q	non-dimensional generalized coordinates (with respect to time t)
q_{dyn}	dynamic pressure
V	true airspeed
V_E	equivalent airspeed
V_p	matrix of eigenvectors
V_0	reference true airspeed
η	generalized coordinates
λ	vector of eigenvalues
ν	airspeed ratio
ρ	true air density
ρ_0	reference air density
σ	air density ratio
$\mathbf{\Phi}$	modal transformation matrix
ω	natural frequency

Introduction

The aero-servoelasticity represents the combination of several theories regarding different aspects of aircraft dynamics. Studies of aero-servoelastic interactions on an aircraft are very complex problems to solve, but essential for aircraft certification. The instabilities issued by the adverse interactions among the flexible structure, the aerodynamic forces, and the control laws acting on it could appear at any time inside the flight envelope, so we can state that the aero-

suite de la page 1

généralisées en utilisant les polynômes de Chebyshev et leurs propriétés d'orthogonalité. Une comparaison de cette nouvelle méthode avec la méthode de Padé qui est habituellement utilizée pour calculer l'approximation de forces aérodynamiques généralisées du domaine de la fréquence dans le domaine du Laplace est aussi présentée. Cette comparaison nous montre que cette nouvelle méthode donne des résultats excellents par rapport à la méthode Padé en l'appliquant sur l'Aircraft Test Model (l'ATM) de NASA DFRC (Dryden Flight Research Center).

servoelastic interactions concern mainly the research field located at the intersection of the following three disciplines: aerodynamics, aeroelasticity, and servo-controls. One main aspect of the aero-servoelasticity is the conversion of the unsteady generalized aerodynamic forces **Q**(k, *Mach*) from the frequency domain into the Laplace domain **Q**(s), where k represents the reduced frequency, *Mach* is the Mach number, and s is the Laplace variable. There are mainly three classical methods for approximating the unsteady generalized forces by rational functions from the frequency domain to the Laplace domain (Tiffany and Adams, 1984, 1988; Edwards, 1977; Roger, 1977; Karpel, 1982): Least Square (LS), Matrix Padé (MP), and Minimum State (MS). All three methods use rational functions under Padé form.

Several aero-servoelastic analysis software codes are developed for the aerospace industry. One of the computer programs used for aero-servoelasticity analyses is the Analog and Digital Aeroservoelasticity Method (ADAM) that was developed at The Flight Dynamics Laboratory (Noll et al., 1986). ISAC (The Interaction of Structures, Aerodynamics, and Controls) was developed at NASA Langley Research Center (Adamsand Hoadley, 1993). At the Boeing Company, the aero-servoelastic software FAMUSS was used (Pitt, 1992). An aeroelastic code, ZAERO, has been developed at Zona Technology, which has been used for aero-servoelastic studies (Chen and Sulaeman, 2003). The STARS code was developed at NASA Dryden Flight Research Center (Gupta, 1997).

Among all these computer codes, we have chosen to work with the STARS code. The STARS program is an efficient tool for aero-servoelastic interactions studies and has an interface with NASTRAN (Newsom et al., 1984; Rodden et al., 1979) a computer program frequently used in the aeronautical industry. In this paper, the lateral dynamics of a half aircraft test model ATM is modeled in STARS. Following finite element modeling and the doublet lattice method application on the ATM in STARS, the unsteady aerodynamic forces are calculated as functions of reduced frequencies k and Mach number M. We have provided here a bibliographical research on the software used in aero-servoelasticity. All these codes exploit two main classical methods for aerodynamic-force approximations from the frequency domain (aeroelasticity) into the Laplace domain (aeroservoelasticity): Least Squares (LS) and Minimum State (MS).

We will now present in detail bibliographical research on other existing methods in the literature.

The approximation of the unsteady generalized aerodynamic forces is a must for the control analysis of our system. Due to the fact that **Q**(k, *Mach*) can only be tabulated for a finite set of reduced frequencies, at a fixed Mach number M, it must be interpolated in the s domain to obtain **Q**(s). In this paper, we describe such an interpolation method that uses the Chebyshev polynomials and its results. In the subsonic regime, the unsteady generalized aerodynamic forces **Q**(k, *Mach*) are calculated, using finite elements computer programs such as STARS or NASTRAN, by the Doublet Lattice Method (DLM). We further need to convert these forces into the Laplace domain where they will be denoted as **Q**(s). The aerodynamic forces dependence of s may be written as an irrational function even for simple cases such as two-dimensional potential incompressible flows on an airplane wing profile. Theodorsen (1933) proved that **Q**(s) could be expressed by use of Hankel's functions. A few years later, Wagner found the first rational approximation (Dowell, 1995) for **Q**(s). Another approach used the approximations of unsteady aerodynamic forces by Padé polynomials. This approach was based on a fractional approximation of the form $P(s)/R(s)$, where P and R are two polynomials in s, for every term of the unsteady force matrix. In this way, every pole of $R(s)$ showed a new state called the augmented state, in the final linear invariant aero-servoelastic system. Thus, if the initial square matrix had N dimensions, and if a Padé approximation of M order is used, then there will be introduced $N(N + M)$ augmented states. The number of augmented states was reduced by Roger (1977). In his formulation, only $N \times M$ modes were introduced, where N is the number of initial modes. Roger's method is based on the fact that the aerodynamic lag terms remain the same for each element of the unsteady aerodynamic force matrix. Thisis called the LS method and is used in computer aero-servoelastic codes such as STARS and ADAM.

Another method was derived from the LS method and was proposed by Vepa (1977). This method used the same denominators for every column of the aerodynamic matrix **Q**, and was called MP method.

Various improvements were made to the two methods presented above. One such type of improvement is that different conditions (restrictions) were imposed on approximations to pass through certain points. Generally, the approximations were restricted to be exact approximations at zero and at two other chosen points. Generally, the first point was chosen to represent the estimated flutter frequency and the second point to represent the gust frequency. Then, the improved methods were renamed: ELS method (Extended Least Squares) (Tiffany and Adams, 1984) and EMMP method (Extended Modified Matrix Padé) (Dunn, 1980). Later, Karpel (1998) proposed a completely different approach to solve the above approximations. He knew from the beginning that the goal was to find a linear invariant system in the time domain and he decided to integrate this information directly in the equation giving the unsteady aerodynamic forces values, by adding a term resembling a transfer function of a linear system. Further, as he wanted to find a linear system of reasonable dimensions, he wrote the approximation under the MS form. The advantage of this method with respect to Roger's method resided in the fact that it allowed an excellent approximation to be obtained, but with a smaller number of augmented states. All of the methods described above allow for the approximation of unsteady aerodynamic forces for one Mach number at a time. To obtain approximations for several Mach numbers, we should perform the approximation approach given by the MS method for each Mach number, which might be expensive in terms of computing time. A valid approximation for a range of Mach numbers could be useful for military Fly-By-Wire aircraft, where the Mach number varies rapidly during high-speed manoeuvres, and where aero-servoelastic interactions are extremely important. Poirion (1995, 1996) constructed an approximation allowing for the calculation of unsteady aerodynamic forces for Mach number values contained in a specific interval and for a frequency domain. He used the MS method and considered a regular dependence with the Mach number. He used several MS approximations, obtained for several fixed Mach numbers, and a spline interpolation method for Mach number dependence. Thus, he obtained a formula that allows for the computing of unsteady aerodynamic forces for any couple (k, *Mach*), where k is the reduced frequency, the equations remaining valid for a range of (k, *Mach*) $\in [k_{\min}, k_{\max}]*[M_{\min}, M_{\max}]$.

The approximation methods should satisfy two opposed criteria simultaneously: an excellent (exact) approximation, which could be obtained by increasing the number of lag terms and a linear invariant system in the time domain of a very small dimension (with the smallest possible number of lag terms). There was no method satisfying both criteria until now. In two recent papers, Cotoi and Botez (2001, 2002) have proposed a new approach based again on a precise Padé approximation. The two authors used order-reduction methods for the last term of the approximation, which could be seen as a transfer function of a linear system. The approximation error for this new method is 12–40 times lower than for the MS method for the same number of augmented states and depends of the choice made for the model reduction method. However, this method remains very expensive in terms of computing time. Dinu et al. (2005) presented open-loop flutter analysis results using a new method based on Chebyshev polynomials theories of the ATM (Aircraft Test Model). Control laws were not considered in their previous paper (where open-loop flutter analysis was performed), while in the present paper, this new method is applied to the ATM— on which ailerons and rudder act — so that closed-loop flutter analysis results are determined.

Aircraft Equations of Motion

The flexible aircraft equations of motion, where no external forces are included, may be written in the time-domain as follows:

$$\tilde{\mathbf{M}}\ddot{\eta} + \tilde{\mathbf{C}}\dot{\eta} + \tilde{\mathbf{K}}\eta + q_{\text{dyn}}\mathbf{Q}(k, Mach)\eta = 0 \tag{1}$$

where ρ is the air density, V is the true airspeed, and $q_{\text{dyn}} = 0.5\rho V^2$ is the dynamic pressure; η is the generalized coordinates variable defined as $q = \mathbf{\Phi}_\eta$ where q is the displacement vector and $\mathbf{\Phi}$ is the matrix containing the eigenvectors of the following free-vibration problem:

$$\mathbf{M}\ddot{q} + \mathbf{K}q = 0 \tag{2}$$

The following transformations were used in Equation (1):

$$\tilde{\mathbf{M}} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}, \qquad \tilde{\mathbf{C}} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\,\mathbf{\Phi}, \qquad \tilde{\mathbf{K}} = \mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\,\mathbf{\Phi}$$

$$\mathbf{Q}(k, Mach) = \mathbf{\Phi}^{\mathrm{T}} A_{\mathrm{e}}(k)\mathbf{\Phi} \tag{3}$$

Here, **M**, **K**, and **C** are the generalized mass, stiffness, and damping matrices; k, the reduced frequency is written as $k = \omega b/V$ where ω is the natural frequency and b is the wing semi-chord length. $A_{\mathrm{e}}(k)$ is the aerodynamic influence coefficient matrix for a given fixed Mach number M and a set of reduced frequencies values k. The Laplace transformation is further applied to Equation (1), and we obtain:

$$[\tilde{\mathbf{M}}s^2 + \tilde{\mathbf{C}}s + \tilde{\mathbf{K}}]\eta(s) + q_{\text{dyn}}\mathbf{Q}(s)\eta(s) = 0 \tag{4}$$

$\mathbf{Q}(s)$ are the unsteady aerodynamic force approximations of $\mathbf{Q}(k, Mach)$ in the Laplace domain. In this paper, we describe a new approximation method that uses Chebyshev polynomials and its results.

Chebyshev Polynomial Theory

These polynomials (Rivlin, 1990; Weisstein, 1999–2005) are a set of orthogonal polynomials defined as the solutions to the Chebyshev differential Equation (10) and are denoted as $T_n(x)$. They are used as an approximation to a least squares fit, and are closely connected to trigonometric multiple-angle equations. The Chebyshev polynomials of the first kind denoted by $T_n(x)$ are implemented in Mathematica as ChebyshevT [n, x], and are normalized so that $T_n(1) = 1$.

Any continuous function may be expressed by use of Chebyshev polynomials using the following equation:

$$f(x) = \frac{1}{2}c_0 + \sum_{j=1}^{\infty} c_j T_j(x) \tag{5}$$

where Chebyshev polynomials used in Equation (5) are expressed under the following form:

$$T_j(x) = \cos(j \arccos(x)) \tag{6}$$

and the coefficients c_j used in Equation (5) are expressed as follows:

$$c_j = \frac{2}{\pi}\int_{-1}^{1} \frac{f(x)T_j(x)}{\sqrt{1-x^2}}\,\mathrm{d}x \quad \text{and} \quad j = 1, 2, \ldots \tag{7}$$

The Chebyshev polynomials have orthogonality properties that allow us to keep the approximation's error within a predetermined bandwidth.

The following recurrence relationships are used in the new approximation method:

$$\begin{cases} T_0(x) = 1 \\ T_1(x) = x \\ T_{r+1}(x) = 2xT_r(x) - T_{r-1}(x) \end{cases} \tag{8}$$

and the following condition in the aim to find the Chebyshev polynomials solution is imposed:

$$T_r(x) = 0 \tag{9}$$

where r specifies the rank of the Chebyshev polynomial. Equation (9) gives the following solution:

$$x = \cos\frac{(2j+1)\pi}{2r} \tag{10}$$

$T_r(x)$ is a function defined by cosines, which lets us conclude that between two solutions of this function we find an extreme of |1| amplitude in the middle of the interval, specifically at:

$$x = \cos\frac{j\pi}{r}, \quad j = 0, 1, \ldots, r \tag{11}$$

Methodology for the Chebyshev Approximation Method

To develop our approximation method, we used the predefined functions for the Chebyshev polynomials expressed in Equation (6) that have been implemented in the Maple kernel, in MATLAB.

These functions (*chebpade* and *chebyshev*) allow the construction of a polynomial interpolation for the unsteady generalized aerodynamic forces, acting on the ATM for 14 values of reduced frequencies k and 9 values of Mach number. The elements forming the matrices of the unsteady generalized aerodynamic forces calculated by the Doublet Lattice Method DLM in STARS are denoted by $\mathbf{Q}(i, j)$ with $i = 1\ldots8$ and $j = 1\ldots8$ for the first eight elastic modes.

The approximation by means of this method is obtained by use of a similar path to the one used for the Padé method. For each element of the unsteady aerodynamic force matrix, we found a power series development under the following form, by use of Maple's "*chebyshev*" function:

$$\mathbf{Q}_{ij}(s) = \frac{1}{2}c_0^{(ij)} + \sum_{n=1}^{\infty} c_n^{(ij)} T_n^{(ij)}(s) \tag{12}$$

where

$$c_n^{(ij)} = \frac{2}{\pi}\int_{-1}^{1} \frac{\mathbf{Q}_{ij}(s)T_n^{(ij)}(s)}{\sqrt{(1-s^2)}}\,\mathrm{d}s \quad \text{for} \quad n = 0, 1, \ldots$$

We found an approximation by rational fractions by use of the "*chebpade*" function:

$$\hat{\mathbf{Q}}_{ij}(s) = \frac{\sum_{n=0}^{M} a_n^{(ij)} T_n^{(ij)}(s)}{1 + \sum_{n=1}^{P} b_n^{(ij)} T_n^{(ij)}(s)} \tag{13}$$

where $M = P + 2$.

This new form integrates the orthogonality properties of Chebyshev polynomials and allows the variation of the degree of the numerator M and the denominator P, to obtain a very good approximation.

In Equation (13), an approximation order $[M, P] = [16, 14]$ gives $M = 16$ and $P = 14$ where M is the maximum rank of Chebyshev polynomials at the numerator and P is the maximum rank of Chebyshev polynomials at the denominator.

We compared the results found by means of our Chebyshev approximation method with the results given by the Padé method. These results are expressed in terms of a total normalized approximation error.

The Padé method uses a parameter identification solution to determine a polynomial fractional form that identifies an orthogonal polynomial interpolation. This fractional form is the key aspect of this method, due to the fact that it allows the order reduction system.

The Padé polynomials are used in the LS method of implementation, which is considered the *most classical* and *most used* method now for aero-servoelastic interactions studies. The LS method was implemented in most aero-servoelasticity software ISAC, ADAM, and STARS.

For various aircraft types (such as CL-604 or F/A-18), classical aero-servoelasticity studies by use of the LS or MS methods were performed. Following an analysis of the results obtained and the algorithms used, we found that the LS method was easier to implement and execution time was faster than that of the MS method. Our computer programs were written in MATLAB. For this reason, in this paper, a comparison is performed between flutter results obtained with the Padé method and flutter results obtained with our method based on Chebyshev polynomials properties.

Figures 1 and **2** show the real and imaginary parts of aerodynamic forces elements approximated by Padé and Chebyshev methods versus their initial values calculated in STARS in the frequency domain. **Figures 3** and **4** show that our new approximation method gives the best approximation error on an interval chosen in the proximity of each approximation point. The effect of the Chebyshev polynomials properties is seen in these types of results. Due to these properties, we are able to impose a bandwidth for the error convergence for each element of the unsteady generalized aerodynamic force matrices.

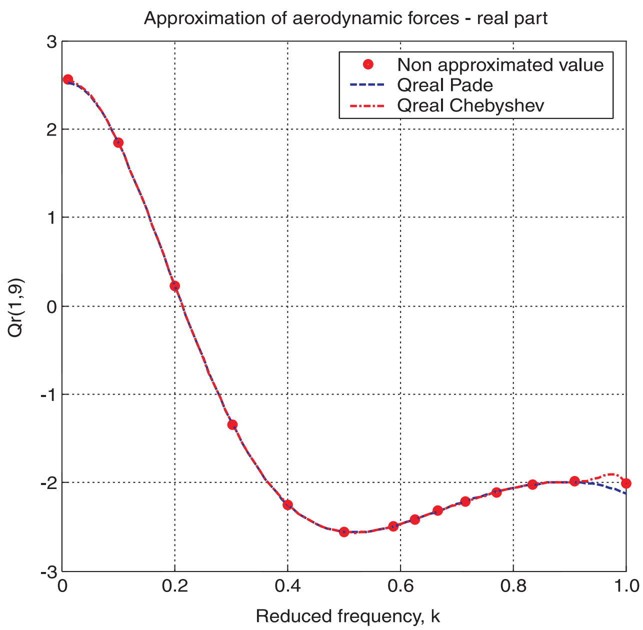

Figure 1. The real part of the aerodynamic forces calculated by Padé and Chebyshev methods versus their initial values in the frequency domain.

Both figures show the overall normalized approximation error by Chebyshev and Padé methods. The differences between **Figures 3** and **4** are based on the model order: **Figure 3** shows the results for the [16, 14] polynomial model order while **Figure 4** shows the results obtained for the [15, 13] polynomial model order. In these examples, the results were obtained using the ATM data generated in the STARS code at Mach number M = 0.5 and for 14 reduced frequencies k = [0.0100 0.1000 0.2000 0.3030 0.4000 0.5000 0.5882 0.6250 0.6667 0.7143 0.7692 0.8333 0.9091 1.0000].

The Padé method gives a small error near the middle of the approximation interval and an increased error towards each end of it. The Chebyshev approximation method demonstrates an almost constant value of the error all along the approximation interval. The total normalized approximation errors differences at both ends of the approximation interval are noticed in both figures. Those differences are higher for k_{14} = 1 than for k_1 = 0.01.

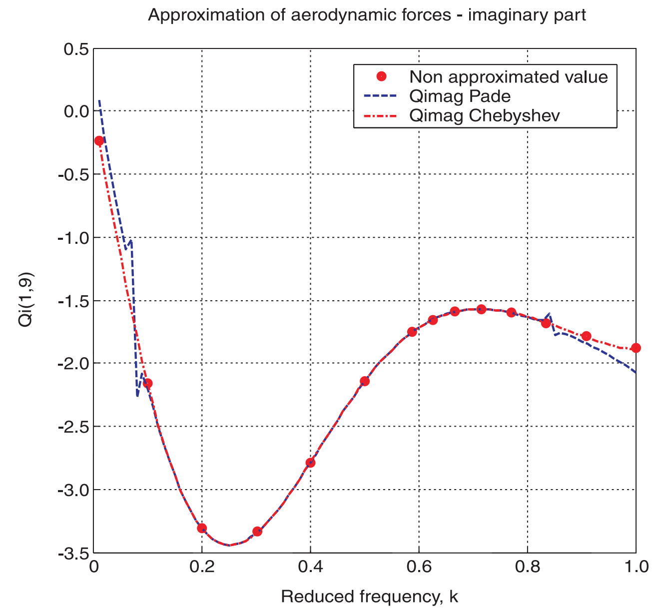

Figure 2. The imaginary part of the aerodynamic forces calculated by Padé and Chebyshev methods versus their initial values in the frequency domain.

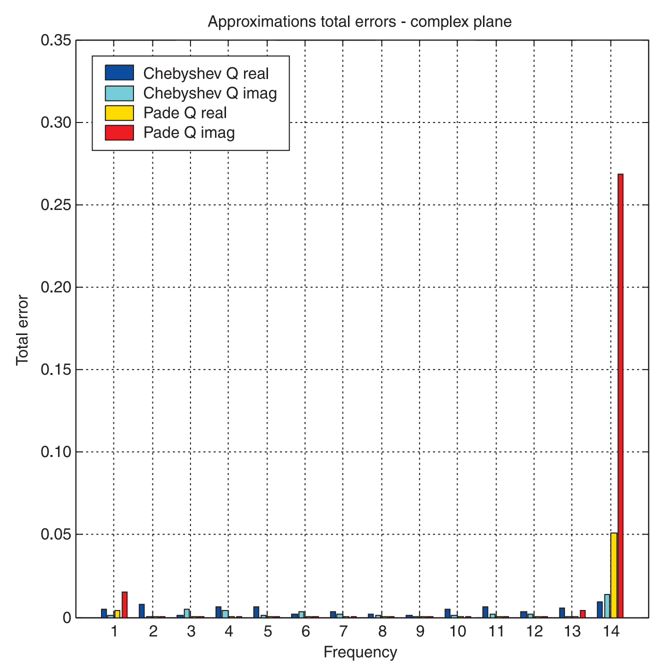

Figure 3. The total normalized approximation errors for the [16, 14] model order.

The total normalized approximation errors were calculated for different values of the polynomial approximation order using the Padé and the Chebyshev polynomial fraction methods (polynomial model order should be equivalent for both methods) for all the Mach numbers and the same differences were observed. The total normalized approximation error obtained with the Chebyshev method was found to be much

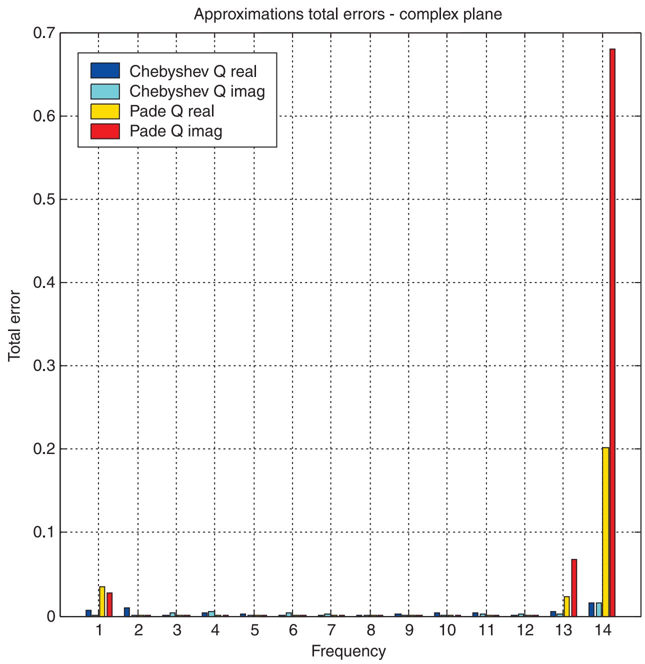

Figure 4. The total normalized approximation errors for the [15, 13] model order.

smaller with respect to the total normalized approximation error given by the Padé method.

In **Table 1**, a few examples regarding the values of the total normalized approximation errors provided by the Chebyshev method and by the Padé method are given. In **Table 1**, three different orders of approximation were used: [16, 14], [15, 13], [10, 8] for the same fight condition and the results are presented for the real aerodynamic forces part errors denoted by $J_{\mathbf{Q}\_\mathrm{REAL}}$ and for the imaginary aerodynamic forces part errors denoted by $J_{\mathbf{Q}\_\mathrm{IMAG}}$. It can be clearly seen that no matter the order of the polynomial approximation, the total normalized error for the Chebyshev method is lower than the total normalized error given by Padé method. These errors were calculated using the following formula:

$$J_{\mathbf{Q}\_\mathrm{REAL}} = \sum_{k=1}^{14}\left(\sum_{i=1}^{N_{\mathrm{modes}}}\left(\sum_{j=1}^{N_{\mathrm{modes}}}\frac{\left|\mathbf{Q}_{ij\,\mathbf{R}\mathrm{new}} - \mathbf{Q}_{ij\,\mathbf{R}\mathrm{old}}\right|}{\sqrt{\left|\mathbf{Q}_{ij}\right|^2}}\right)\right)\times 100\% \quad (14)$$

$$J_{\mathbf{Q}\_\mathrm{IMAG}} = \sum_{k=1}^{14}\left(\sum_{i=1}^{N_{\mathrm{modes}}}\left(\sum_{j=1}^{N_{\mathrm{modes}}}\frac{\left|\mathbf{Q}_{ij\,\mathbf{I}\mathrm{new}} - \mathbf{Q}_{ij\,\mathbf{I}\mathrm{old}}\right|}{\sqrt{\left|\mathbf{Q}_{ij}\right|^2}}\right)\right)\times 100\%$$

where $\mathbf{Q}_{\mathbf{R}\mathrm{old}}$ and $\mathbf{Q}_{\mathbf{I}\mathrm{old}}$ are the real and the imaginary parts of the unsteady aerodynamic forces given by STARS for the ATM model and $\mathbf{Q}_{\mathbf{R}\mathrm{new}}$ and $\mathbf{Q}_{\mathbf{I}\mathrm{new}}$ are the real and the imaginary parts of the unsteady aerodynamic forces approximated by

Table 1. Total normalized approximation errors by Chebyshev and Padé methods.

Order	Method	$J_{\mathbf{Q}\_\mathrm{REAL}}$	$J_{\mathbf{Q}\_\mathrm{IMAG}}$
[16, 14]	Chebyshev	0.065065	0.037611
	Padé	0.065271	0.287657
[15, 13]	Chebyshev	0.055391	0.040053
	Padé	0.260147	0.775989
[10, 8]	Chebyshev	0.061466	0.035109
	Padé	0.136338	0.055638

Chebyshev or by Padé theories. N_{modes} is the total number of modes (also the dimension of **Q**), k is the index of the reduced frequency, and J is the total normalized error.

Open-Loop Flutter Results Obtained using the Chebyshev Method

To validate our method, we used the STARS ATM developed by the NASA Dryden Flight Research Center. This lateral model (only anti-symmetric modes are provided) includes aero-structural elements (flexible aircraft) and control surfaces (ailerons and elevator). First, a free vibration analysis was performed in the absence of aerodynamics to obtain the free modes of vibration. We obtained the same frequencies and modes of vibration using our MATLAB method as with STARS.

Then to calculate aerodynamic forces in the frequency domain by the DLM, the same simulation parameters were considered as the ones considered in the STARS computer program: reference semi-chord length b = 38.89 in (1 in = 2.54 cm), reference air density at sea level ρ_0 = 1.225 kg/m^3, altitude at sea level Z = 0 ft (1 ft = 0.3048 m), reference sound airspeed at sea level a_0 = 340.294 m/s.

In **Table 2**, the speeds and frequencies at which flutter occurs are calculated by two methods, Padé and Chebyshev, for three different types of approximation orders. The execution speed is three times smaller for the Chebyshev polynomials method than the execution speed used for the Padé method.

Table 2. Flutter results comparison for ATM in open loop.

	Flutter results (fuselage first bending mode)		
Method	Speed (knots)	Frequency (rad/s)	Computation time (s)
Pk-Padé [8, 6]	445.5	77.5	122
Pk-Padé [9, 7]	445.5	77.5	134
Pk-Padé [10, 8]	445.8	77.5	144
Pk-Chebyshev [8, 6]	446.5	77.5	40
Pk-Chebyshev [9, 7]	446.6	77.5	47
Pk-Chebyshev [10, 8]	446.6	77.5	53

Closed-Loop Flutter Results Obtained using the Chebyshev Method

We applied our new approximation method for the closed-loop aero-servoelastic analysis, using the transfer function information provided by the aircraft's control laws (ailerons and elevator are used for lateral aircraft control). The conversion from the frequency domain into the Laplace domain (as closed-loop calculations should be realized in the Laplace domain) was done this time by use of the LS method for the approximations obtained with Chebyshev polynomials. To achieve this type of conversion, we used the following variable change:

$$s = \mathrm{j}k \Rightarrow k = \frac{s}{\mathrm{j}} = -\mathrm{j}s \tag{15}$$

where "j" is the complex number $\mathrm{j} = \sqrt{-1}$.

We rewrote the approximation of the unsteady generalized force matrix in the Laplace domain as follows:

$$\mathbf{Q}(s) = A_0 + A_1(-\mathrm{j}s) + A_2(-\mathrm{j}s)^2 + A_3 \frac{-\mathrm{j}s}{-\mathrm{j}s + b_1} + A_4 \frac{-\mathrm{j}s}{-\mathrm{j}s + b_2} + \cdots \tag{16}$$

and we took into account that $\mathbf{Q}(s)$ has a real part $\mathbf{Q}_\mathrm{R}(s)$ and an imaginary part $\mathbf{Q}_\mathrm{I}(s)$, such as:

$$\mathbf{Q}(s) = \mathbf{Q}_\mathrm{R}(s) + \mathrm{j}\,\mathbf{Q}_\mathrm{I}(s) \tag{17}$$

We obtained then the following expressions for these aerodynamic forces:

$$\begin{cases} \mathbf{Q}_\mathrm{R}(s) = A_0 + s^2 A_2 + \sum_{n=1}^{\infty} \frac{s^2}{s^2 + b_n^2} A_{n+2} \\ \mathbf{Q}_\mathrm{I}(s) = -A_1 s - \sum_{n=1}^{\infty} \frac{s b_n}{s^2 + b_n^2} A_{n+2} \end{cases} \tag{18}$$

where $A_0, A_1, \ldots, A_{n+2}$ are the LS decomposition matrices and b_n are the lag terms introduced by the LS method.

Figures 5, **6**, and **7** show the flutter closed-loop results. In **Figure 5**, the frequency is plotted versus the damping, in **Figure 6**, the frequency is plotted versus the Equivalent Airspeed and in **Figure 7**, and damping is plotted versus the Equivalent Airspeed. From these three figures, the flutter frequency, damping, and equivalent airspeed are calculated (flutter occurs where damping is zero). Their numerical values are given in **Table 3**.

In **Table 3**, a comparison of the results obtained with our new method (a combination of the *P* flutter method with the Chebyshev approximations) with the results obtained by a combination of the *P* flutter method with the Padé approximations is provided for 2 to 6 lag terms.

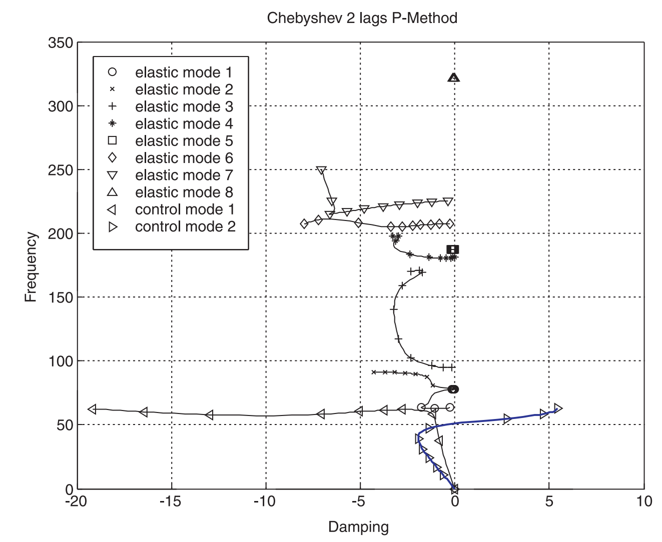

Figure 5. Frequency versus damping calculated by the *P*-Chebyshev method for the closed loop ATM analysis.

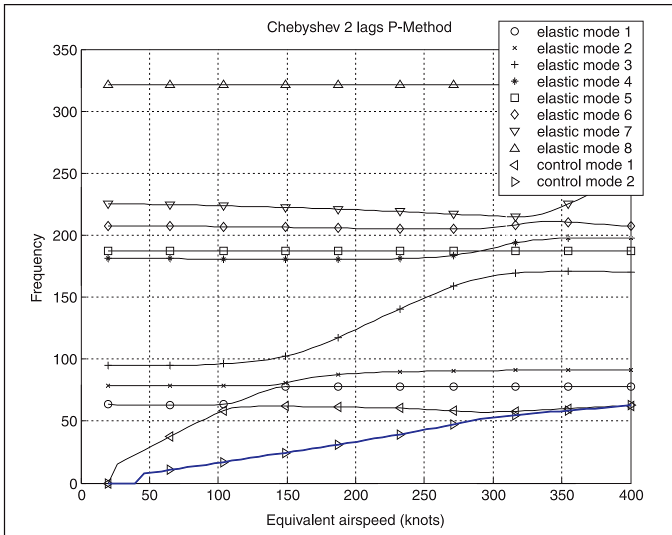

Figure 6. Frequency versus equivalent airspeed calculated using the *P*-Chebyshev method for the closed loop ATM analysis.

Our new approximation method provides the same values for the flutter speeds and frequencies as the classical methods (which takes Padé approximations into account, because these methods are actually based on Padé) no matter the number of lag terms considered. These good results were expected to occur, following the properties of the Chebyshev polynomials. In addition, the execution time required by the *P-Chebyshev* method is three times smaller than the execution time required by the *P-Padé* method.

It is well known that a larger number of lags imply on one hand an increased number of calculations and on the other hand it introduces new poles, thus modifying the initial system and

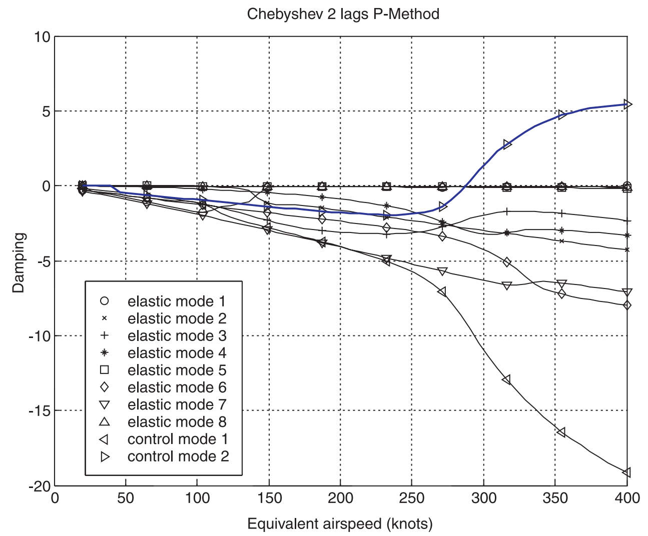

Figure 7. Damping versus equivalent airspeed calculated using the *P*-Chebyshev method for the closed loop ATM analysis.

Table 3. Flutter results comparison for ATM in closed loop.

	Flutter results (control mode 2)		
Method	Speed (knots)	Frequency (rad/s)	Computation time (s)
P-Padé [4, 2]	287.7	51.15	354
P-Padé [5, 3]	287.6	51.13	361
P-Padé [8, 6]	287.6	51.10	392
P-Chebyshev [4, 2]	287.6	50.92	97
P-Chebyshev [5, 3]	287.6	50.92	102
P-Chebyshev [8, 6]	286.7	50.72	126

generating approximation errors. For this reason, the advantage of using this new method with respect to classical methods is that using a small number of lags (2 lag terms) the same results are obtained as if using a large number of lags (6 lag terms).

Conclusion

The Chebyshev approximation method provides excellent flutter results for a small number of lag terms. However, due to the fact that the Chebyshev polynomials were generated by use of the ATM data, there are quite large differences between the values of the elements contained in the unsteady generalized aerodynamic force matrices (1e +10). Restraints regarding the threshold of the approximation error had to be imposed, i.e., for smaller elements we imposed an error value of 1e –4 and for larger elements an error value of 1e –2. Without these restraints, the Chebyshev polynomials cannot be generated.

We could see that by use of the Chebyshev method in open loop, we were able to find very good values for the flutter speeds and the frequencies at which flutter occurs. One of the most important achievements of our new method, if not the most important, is the fact that the computation time for the open-loop case is up to 3 times smaller than in the *Pk*–Padé method and up to 30 times smaller than in our *Pk*–LS case, even for an increased approximation order.

As for the closed-loop case, we observed that no matter the initial Chebyshev approximation order, it would be enough to make use of only 2 lags when converting to the Laplace domain to obtain excellent approximation results.

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