Pattern Spectrum as a Local Shape Factor for Off-Line Signature Verification

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Abstract

A fundamental problem in the field of off-line signature verification is the lack of a pertinent shape representation or shape factor. The main difficulty in the definition of pertinent features lies in the local variability of the signature line which is closely related to the intrinsic characteristic of human beings. In this paper we proposed a new formalism for signature representation based on visual perception. A signature image of 512x128 pixels is centered onto a grid of rectangular retinas which are excited by a local portion of the signature image. So each retina has only a local perception of the entire scene. Granulometric size distributions have been used for the definition of local shape descriptors in attempt to characterized the amount of signal activity in front of each retina located on the focus of attention grid. Experimental evaluation of this scheme has been made using a signature database of 800 genuine signatures from 20 individuals. Two types of classifiers, a 1NN and a threshold classifiers show a total error rate below 0.02% and 1.0% respectively in the context of random forgeries.

1. Introduction

In the field of pattern recognition, off-line signature verification still an open problem [4]. Making abstraction of the difficult task related to the discrimination of freehand or simulated forgeries based on the analysis of the signature image only, the complete elimination of random forgeries define as genuine signatures of other writers enrolled to the verification system is again a goal to reach. A short analysis of this problem reveals that this task is very easy for human beings. Consequently, why does this pattern recognition task seems so much difficult to be automated successfully ?

A survey of the principal schemes in the literature suggested that a new way of addressing the problem of signature verification be formulated in order to find a satisfactory solution for eliminating random forgeries. A possible answer to the verification problem could be stated from a deep analysis of the intrinsic characteristics of genuine signatures; handwritten signature which is a special case of handwriting is the result of a rapid movement. The immediate consequence of this fact is that the overall morphology of the signature is relatively stable over time when the signature is place on a pre-establish context like this one of bank cheques ; that is to say a staff line surrounded by printed text which define a spatial constraint for the signer. Despite this characteristic of the handwritten signature, a fundamental problem in the field of off-line signature verification still the lack of a pertinent shape representation or *shape factor*. The main difficulty in the definition of pertinent features lies in the local variability of the signature line which is closely related to the intrinsic characteristic of human beings [3,5]. How the local variability of the signature could be take into account in the definition of a shape descriptor tailor made for verification purposes? This is a very difficult task to do [8] and this is why a new approach is needed.

2. Formal model for shape factor definition

We proposed in this paper a new formalism for the definition of a signature representation based on visual perception. Based on observations made on the superposition of genuine signatures shown in Figure 1, the following assumptions can be stated in a way to define a shape descriptor tailor made for the signature verification problem :

1) *the overall orientation and the aspect ratio of genuine signatures written in a constrained 2D area are relatively stable for each writer, and*
2) *the local variability of the signature line is an intrinsic characteristic of the identity of the writer and it should also be take into account. This phenomenon*

is characterized by local displacements of strokes following the principal axis of the signature.

Following these assumptions, the invariance in rotation and in scale are not required in the definition of a shape descriptor adapted for the verification task and only a correction in translation is necessary. The originality of the proposed approach is that local measurements $m(\sphericalangle)$ are not made on specific parts of the signature (primitives) but on specific areas in the image plane located to several focus of attention in the scene. In this way, the identification and the segmentation of feature points or primitives is overcome.

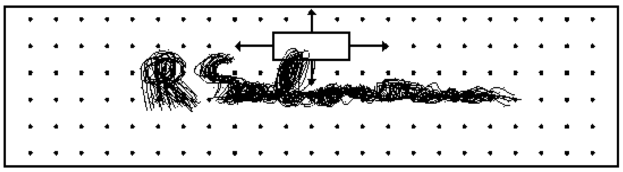


Figure 1

The method proposed for feature extraction is simple. A signature image of 512*x*128 pixels is centered on a *grid of rectangular retinas* which are excited by local portions of the signature image [Figure 1]. So each retina has only a local perception of the entire scene and the measurement $m(\sphericalangle)$ make on the subset X of pixels related to a retina will reflect the local activity of the signal. An important aspect to be considered in the definition of a shape factor tailor made for the signature verification problem is that the systematic absence of signal activity in specific areas in the image will be take into account in attempt to characterized the shape of the signature. This is achieved by assumption 1 which stated that the overall proportion of genuine signatures is relatively stable. In other words, not only the signature itself but the background of the signature image is also considered in the definition of the shape descriptor. Following assumption 2, the local variability of the signature line could be take into account easily by the addition of a certain percentage of (horizontal and/or vertical) overlap between neighboring retinas. As the visual observations made on genuine signatures showed that in general the displacement of strokes follows the principal axis of the signature, this fact suggest that only horizontal overlap between neighboring retinas will be enough.

The measurement $m(\sphericalangle)$ applied to the set X of pixels related to a retina should be able to detect the presence or the absence of any signal activity and it should be able in some way to quantify it. Several types of transformation can be used for the implementation of $m(\sphericalangle)$. The morphological operations of opening and closing seem useful for this kind of task. The *pattern spectrum* is an internal morphological shape descriptor called *pecstrum* [6]. The pecstrum is computed by measuring the results of successive morphological openings of the object by a structuring element that increases in size. Our objective was to extend this approach of granulometric size distributions to the definition of local shape descriptor in attempt to characterized the amount of signal activity in front of each retina located on the focus of attention grid [Figure 1].

3. Local granulometric size distributions as a shape factor

3.1 Basic definitions of morphological operators

Mathematical morphology is a set-theoretical approach for the analysis of geometrical structures. Erosion and dilation are the basic morphological operations [10]. The *eroded* set of X with respect to B and denoted by $X \mathbf{S} B^S$ is represented by all points $z \in E$ for which the translates B_Z are totally included in X:

$$X \mathbf{S} B^S = \{z \in E : B_Z \subset X\} \qquad (1)$$

where B^S is a rotated version of the *structuring element* SE by 180° on the plane E and where $\mathbf{S}$ denotes the Minkowski subtraction. The *dilation* of X with respect to B is composed of all points $z \in E$ for which the translates B_Z hit X:

$$X \oplus B^S = \{z \in E : B_Z \cap X \neq \varnothing\} \qquad (2)$$

where $\oplus$ denotes the Minkowski addition. An erosion followed by a dilation gives a new morphological operation called *opening* of X with respect to B, denoted by X_B which is defined as the union of all the translates B_Z of B that lie inside X:

$$(X \mathbf{S} B^S) \oplus B = X_B = \bigcup \{B_Z : B_Z \subset X\}. \qquad (3)$$

Finally, the operation of *closing* X^B is the result of a dilation followed by an erosion which is the dual operation of opening because it can be implemented by opening the complement of X:

$$(X \oplus B^S) \mathbf{S} B = X^B = [(X^c)_B]^c. \qquad (4)$$

A *granulometry* is the result of a set of openings X_{rB} of the object X with an structuring element *rB*, with $r \in \Re \rightarrow X_{rB}$. More formally, the transformation of a set X using transform $\Psi(\cdot)$ and a measure $m(\cdot)$ on it, like $X \rightarrow \Psi(X) \rightarrow m(\Psi(X))$ is a granulometry [10].

The *pattern spectrum* is the result of successive applications of morphological operators on a set using an increasing structuring element *rB*. Opening a signal with a structuring element of size *r* can be viewed as removing details smaller than rB from the signal. The digital version of the pattern spectrum is defined as follows [1] :

$$p(n) = \frac{m(X_{nB}) - m(X_{(n+1)B})}{m(X)}, \quad n = 0,1,\ldots,k-1. \qquad (5)$$

with $$\sum_{n=0}^{k} p(n) = 1. \qquad (6)$$

Each $p(n)$ represents the fraction of the total area of X that is rejected by the opening with the SE $(n+1)B$ provided that the opening with nB has already been performed. The SE used is the integer multiple (nB) of a basic prototype B. The value $n=k-1$ is the one for which the binary object disappears after opening it with kB. In other words, a single impulse at $p(n)=1$ means that the binary object totally disappears by the SE $(n+1)B$. The pecstrum is translation and rotation invariant (considering the limit of a square lattice), but it is not scale invariant. The latter property is not critical in our methodology because the granulometries were evaluated on the same type of retinas in the image, that is to say the dimensions of retinas are all the same [Figure 1] leading to a local normalization of the image and the measurement.

The *negative part of the pattern spectrum* is obtained by measuring the change of the area of the object when successive closings are performed on it [1] :

$$p(-n) = \frac{m(X^{nB}) - m(X^{(n-1)B})}{m(X)}, \quad n = 1,\ldots,k. \qquad (7)$$

Following Serra [10], successive closing of the transformed object approximates its convex hull and the final result varies depending on the shape of the SE in use. The negative part of the pattern spectrum relates some normalized measure of the size of holes and cavities of the object. The first components of both the positive and the negative pecstrum contain information about the boundary roughness of the object. The major drawback of the negative spectrum from a computational point of view lies in the difficulty to determine properly the maximum value of n, k [1,2].

The *pseudopecstrum* p_s was introduced by Anastassopoulos in [1] as a solution to the computational problems of the negative part of the pecstrum. The pseudopecstrum p_s of a binary object X is the ordinary positive pecstrum of the complement X^c, relative to the minimum circle which has its center in the center of gravity of X and contains X. The values $p_s(n)$ are normalized to the original area of the object X and the shape of the structural element is assumed to be circular. The evaluation process of p_s terminates when the area of X^c becomes zero. The information content of the pseudopecstrum is not the same as that contained in the negative pecstrum. It obviously contains information about cavities and holes in the object X but also measures if the object is similar to a circle. The authors mentioned that the pseudopecstrum contain some misleading shape information, especially in its first components which are coming from the regions where the circumscribed circle touches the object X.

3.2 Shape factor definition for signature verification

Following the presentation of the concept of shape descriptors tailor made for the signature verification problem based on visual perception [Section 2], the amount of signal activity from a retina could be characterized in many ways using the morphological operators. We need to define more formally each retina as W the set of pixels in the related area covered by the retina located at a specific point of attention in the image [Figure 2]. Let X be the set of pixels belonging to the signal which activates the retina. As example, the set X showed in Figure 2 is defined by four parts of a signature in the field of view of a rectangular retina W with $X = \bigcup_{i=1}^{4}\{X_i\}$ and $X \subseteq W$. It is obvious that $X = \varnothing$ when no signal activates the retina. Finally, the set E represents the *augmented set* W used in some experiments in attempt to eliminate some noise characterized by pixels belonging to X and located in the neighborhood of the external frontier of set W. Here, no signal activity is present in the surround of W, that is to say in the area defined by $(E \cap W)^c$.

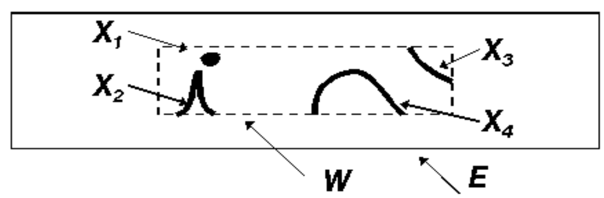
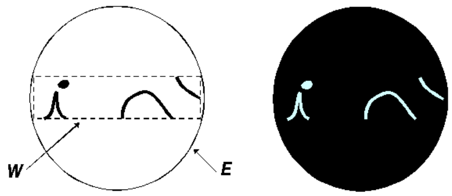


Figure 2

Many alternatives have been evaluated in this work in attempt to define a measurement $m(\cdot)$ of the local activity of signal X in the field of view of a retina W. First, the *positive pecstrum* has been used as a transform $\Psi p(X)$ applied to the set X of signal pixels included in W, with $W = E$ and $(W \cap E)^c = \varnothing$. No bordering effects are anticipated in this case, so it is not necessary to augment the basic retina area. Second, the *pseudopecstrum* $\Psi p_s(X^c)$ has been applied to set X^c, the complement of X in the domain defined by set E, with $W = E$. When the SE used in the realization of transform $\Psi p_s(X^c)$ is one of the segments in the set {◷,|,/,\}, then set E is a rectangular area as depicted in Figure 2 with $W = E$. When a circular SE is used for implementing transform $\Psi p_s(X^c)$, then set E corresponds to the rounded area defined by the circumscribing circle to the set W [Figure 3b]. In both case, transform $\Psi p_s(X^c)$ is applied on the complement of X, i.e. $X^c \subseteq E$, whatever the choice of the SE.



a) b)

Figure 3

Finally, the *augmented pseudopecstrum* $\Psi ap_s(X^c)$ is simply the pseudopecstrum based on a SE in {⌚,|,/,\} and applied to set X^c, the complement of X in the domain defined by set E with $X \subseteq W \subset E$, $(W \cap E)^c \neq \varnothing$ and $X^c \subseteq E$. In this work, a scale factor of 50% in all directions is used in the definition of set E when the augmented pseudopecstrum is applied to E [Figure 2]. As example, a set W related to a rectangular area of 32x16 pixels gives the augmented set E with a cardinality $|E|$ of 64x32 pixels. In the case of a circular set E, the augmentation in area is not necessary because the shape of the SE and this one of set E is the same. So, no bordering effects can occur in this case. Obviously, the field of view of a retina is directly related to its size. This factor has a great impact on the performance in verification of the system.

Table 1: Basic definition of retina *W* (in pixels)

hxv	n_h	n_v	n_r	n
16x16	32	8	256	768
16x32	32	4	128	384
32x16	16	8	128	384
32x32	16	4	64	192
64x32	8	4	32	96

In Table 1, the horizontal h and the vertical v length of a retina (in pixels) permit the direct evaluation of the cardinality $|W|$ of sets W. As the horizontal and vertical dimensions of the signature images are 512 and 128 pixels respectively, the evaluation of the total number of retinas n_r is straightforward and can be evaluated from the product $n_h * n_v$ where n_h and n_v correspond to the number of retinas in the horizontal and the vertical directions respectively. In Table 1, $n_h = 512 \div h$, $n_v = 128 \div v$, and $|W| = h\mathrm{x}v$.

Finally, the shape descriptor of a signature ♍$_i$ is a feature vector of dimension n equal to the number of local descriptors (LD) evaluated from a retina times n_r the number of retinas applied to the entire image. Here, the granulometric moments have been used as local descriptors resulting to the evaluation of measurements m(·) to a specific transformation $\Psi(\cdot)$ applied to set E with $E \rightarrow \Psi(E) \rightarrow m(\Psi(E))$. So, for each retina, the granulometric moments like the mean, the variance and the skewness (LD=3 in this case) are evaluated from transformation $\Psi(\cdot)$. Consequently the dimension n of the feature space ⌚^{n} is equal to (3 * n_r) as showed in Table 1. A value of n=768 represents a very high feature space and normally this is not a tractable way to go in the context of a standard pattern recognition problem. The reason why a high dimension as this has been used for the signature verification problem given that the number of observations availabled for training is very low, is that a lot of retinas showed the systematic absence of signal activities. For that reason, the high dimension of a feature space is indirectly raised down because a lot of elements in the feature vectors have a fixed value.

4. Experimental protocol and signature database description

The proposed signature verification system has been tested using a standard signature database of 800 images (40 signatures written by 20 individuals) [8,9]. The handwritten signatures were written in a 3x12 cm rectangle, using the same type of writing tool (a Pilot Fineliner with flexible felt tip and black ink) and sheets of white paper. Let ***R*** be the reference database related to the first 20 signatures of each writer (← ***R*** ← = 400), and ***T*** be the test database which is related to the last 20 signatures of each writer with ← ***T*** ← = 400.

Two types of classifier have been implemented in this study: a *Nearest Neighbor classifier with vote* and a *minimum distance classifier*. The former allows an evaluation of the discriminant power of a shape factor (signature representation R(♑)) in the case of the signature database under evaluation: this can be related to a lower limit of the total error rate when the maximum available information is kept in memory. The latter is a more realistic solution for the verification problem, but it requires the evaluation of a comparison threshold ♦$^{(i)}$ for each writer enrolled in the verification system. Let ♍ be the set of all feature vectors related to a specific representation R(♑), with ←♍ ← = 800. The size of each feature vector ♍$_i$ ♌ ♍ varies from N=96 to N=768 [Table 1], following the shape factor definition used for the implementation of each individual classifier.

Let us define the sets required for the performance evaluation of classifiers assigned to each writer *(i)* enrolled in the verification system. Let the set of genuine signatures $S_{ref}^{(i)} \subset R^{(i)}$ required as reference signatures be retained for the minimum distance classifier with threshold $\tau^{(i)}$. The cardinality of this comparison set will be $\left|S_{ref}^{(i)}\right| = N_{ref}$. The comparison signatures are chosen randomly in the reference set $R^{(i)}$ with $\left|R^{(i)}\right| = 20$ and their number was fixed to $N_{ref} = 6$ in this study. Let the learning set $S_{learn}^{(i)} \subset R$ of cardinality $\left|S_{learn}^{(i)}\right| = N_{learn}$ be used for the evaluation of threshold $\tau^{(i)}$ for writer *(i)*. Learning set $S_{learn}^{(i)}$ is defined by genuine signatures of writer *(i)* that have not been chosen for $S_{ref}^{(i)}$ (class ω_1), and by 5 signatures chosen randomly from all sets $R^{(j)}$ for other writers (j) with $1 \le j \le 20, i \ne j$; that is to say, 5x19 random forgeries related to class ω_2. In the case of the *NN* classifier, only the learning set $S_{learn}^{(i)}$ is required, with $N_{learn} = 115$, that is to say, all 20 observations from class ω_1 and the 95 observations from class ω_2 have been taken into account. The set $S_{gen}^{(i)} \subset T$ is used for the evaluation of the performance in generalization of each classifier, with $\left|S_{gen}^{(i)}\right| = N_{gen}$. Set $S_{gen}^{(i)}$ is made up of 20 genuine signatures from the test set $T^{(i)}$ of writer *(i)* (class ω_1), and by 5 signatures chosen randomly from all sets $T^{(j)}$ of other writers (j) with $1 \le j \le 20, i \ne j$; that is to say, 5x19 random forgeries related to class ω_2; consequently the cardinality of all sets $S_{gen}^{(i)}$ is $\left|S_{gen}^{(i)}\right| = N_{gen} = 115$. Thus, a statistical independence of sets $S_{gen}^{(i)}$, $S_{ref}^{(i)}$ and $S_{learn}^{(i)}$ is always observed because initial sets R and T satisfy this property.

5. Experimentation

5.1 Prototyping phase

All representations R(♑) that have been defined in Section 3 were evaluated using a *NN classifier with vote*. For each classifier based on a representation R(♑), the performance of the verification system is reported in terms of Type I (ε_1, false rejection of genuine signatures) and Type II (ε_2, false acceptance of random forgeries) error rates evaluated for the 20 writers. The mean total error rate ε_t of a verification system is expressed in terms of Type I and Type II error rates, that is to say $\varepsilon_t = (\varepsilon_1 + \varepsilon_2)/2$.

In this experiment, the evaluation of several strategies has been made on the basis of the ε_t values. The positive pattern spectrums ✡*p*(*X*) based on SE in {|,🕘,/,\}, the pseudopecstrums ✡$p_s(X^c)$ based on SE {|,🕘,/,\,🕖} and the augmented pseudopecstrums ✡$ap_s(X^c)$ based on SE {|,🕘/,\} have been evaluated using several definitions of retinas *W* and of percentage of overlap between neighboring retinas [Table 1].

Experimental results reveal that better results were obtained based on rectangular windows with the longest side located in the horizontal direction, using the augmented pseudopecstrums ✡$ap_s(X^c)$ based on SE {|,🕘,/,\} and the pseudopecstrum ✡$p_s(X^c)$ based on a circular SE {🕖}. The general trend in the data shows that for the class of signals *X* under study, the strokes can be characterized more efficiently by measurements made on the background area in the field of view of the retina *W*. This is accomplish when a transform $\Psi(\cdot)$ is applied to set X^c. Based on these experimental results, a rectangular retina *W* of 32x16 pixels with 50% of overlap in the horizontal direction has been retained for the final implementation of a signature verification system.

5.2 Evaluation of individual classifiers performance

The previous experiments using the augmented pseudopecstrums ✡$ap_s(X^c)$ based on SE {|,🕘,/,\} and this one using the pseudopecstrum ✡$p_s(X^c)$ based on circular SE {🕖} were therefore repeated 25 times for each signature verification system and the observations of class ω_2, that is to say, the subsets of $S_{learn}^{(i)}$ and $S_{gen}^{(i)}$, were redefined randomly following our standard protocol [8,9]. In the case of the *minimum distance classifier*, the effect, on the global performance of the verification system, of the choice and number of reference signatures (class ω_1) [Section 4] in the definition of sets $S_{ref}^{(i)}$ were also taken into account. The mean performances of the signature verification systems $\overline{\varepsilon_t}$ resulting from the performance evaluation on 25 iterations of representations R(♑) used for both types of classifiers are depicted in following Tables 2 and 3.

Table 2: Experimental results using the augmented pseudopecstrum ✡$ap_s(X^c)$ and the NN classifier

SE	$\bar{♏}$(%) (±♦)(%)	$\bar{♏}$(%) (±♦)(%)	$\bar{♏}$(%) (±♦)(%)
{🕘}	0.000 (0.000)	0.040 (0.037)	0.020 (0.019)
{|}	0.000 (0.000)	0.038 (0.046)	0.019 (0.023)
{/}	0.000 (0.000)	0.036 (0.044)	0.018 (0.022)
{\}	0.000 (0.000)	0.031 (0.039)	0.016 (0.020)

{●}	0.200 (0.141)	0.153 (0.092)	0.177 (0.081)

Table 3: Experimental results using the augmented pseudopecstrum ✡$ap_s(X^c)$ and the minimum distance classifier with N_{ref} = 6.

SE	♏̄(%) (±♦)(%)	♏̄(%) (±♦)(%)	♏̄(%) (±♦)(%)
{🕗}	1.550 (0.548)	0.145 (0.153)	0.848 (0.260)
{|}	2.670 (0.754)	0.480 (0.189)	1.575 (0.367)
{/}	3.010 (0.684)	0.318 (0.203)	1.664 (0.348)
{\}	2.070 (0.832)	0.305 (0.185)	1.188 (0.418)

At first glance, it is clear that verification systems built around *NN* classifiers outperform those based on *minimum distance* classifiers for all representations R(♑) under study. As an example, the mean total error rate $\overline{\varepsilon_t}$ varies in the (0.016% - 0.20%) range of values for the *NN* classifier [Table 2] and in the (0.848% - 1.664%) range of values for the *minimum distance* classifier using N_{ref} = 6 reference signatures [Table 3]. A last comment about the circular SE {🕗}. It was decided to discard transformations ✡$p_s(X^c)$ based on this circular SE because the performance obtained with the NN classifier is not as good than this one obtained with SE {|,🕗,/,\}, and because the evaluation of the pseudopecstrums on big retinas is time consuming.

6. Conclusion

A new approach based on visual perception has been proposed in attempt to define a shape factor tailor made for the signature verification problem in the context of random forgeries. Focusing the attention to fixed positions in the 2D space and restricting the field of view of retinas conduct to a representation of the signature based on a local analysis of the scene without the need to segment the signature line in primitives. Moreover, evaluating the amount of the signal activity using morphological operators applied to the background, i.e. on set X^c, permits in some way the measurement of the local variations of the signature line which is closely related to the identity of the writer. Further works will be oriented toward the dynamic definition of the position and of the size of the retinas based on the context of the scene. Moreover, the cooperation of neighboring retinas will probably lies in a more robust evaluation of the local variations of the signature.

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