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SOFTWARE FOR OPERATIONAL MODAL ANALYSIS AND AUTOMATIC IDENTIFICATION OF MODAL PARAMETERS

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ABSTRACT

In this paper, we present a software for the Operational Modal Analysis (OMA) of vibrating structures in operating conditions. The method used is based on a multivariate autoregressive model, with the model's parameters of the model are estimated by least squares via the computation of the QR factorization, and the modal parameters are identified from the eigen-decomposition of the state matrix. The natural frequencies, damping rates and modes shapes are updated with respect to the model order and are successively constructed on stabilization diagrams with their corresponding confidence intervals. Furthermore, an optimal model order can be automatically selected from the evolution of a factor called the Noise rate Order Factor (NOF) from which the structural modes are automatically distinguished from the spurious ones in order to construct noise-free spectra. After the frequency ranges of interest are selected, the natural frequencies and damping rates are automatically identified. The proposed software is user friendly and the operator can easily determine the accuracy of the modal parameters that are automatically computed. Several experimental applications are described by way of examples.

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1. INTRODUCTION

The identification of structural modal parameters [1] plays an important role in structural health monitoring and machinery vibrations. It is usually conducted in a frequency domain by measuring the transfer function between a vibratory response and a known excitation force [2].

However, in several industrial applications where it is not possible to stop a machine, the forces cannot be measured, and so an Operating Modal Analysis (OMA) must therefore be conducted [3, 4]. Since the environmental forces result from natural excitations, operational modal analysis should deal with the time domain. Several methods, such as the Ibrahim time domain method (ITD) [5], the Least squares complex exponential (LSCE) [6] and ARMA [7], can be used in identifying of modal parameters just from responses. Examples of such industrial applications can be found in bridge monitoring [8], in the identification of mass and damping applied to fluid-structure interactions [9, 10], and in crack detection [11].

To date, OMA has been successfully implemented in commercial software applications. Structural Vibration Solutions A/S has developed ARTeMIS, a modal identification software application for ambient response testing [12]. This software uses the Frequency Domain Decomposition (FDD) [13] as the base tool for the extraction of modal parameters from the spectra of the output-only data. The operational modal analysis of the Bruel & Kjaer A/S [14] dedicated to mechanical machines, also uses the FDD in addition to the Stochastic Subspace Identification (SSI) [15] time method. On the other hand, LMS uses the PolyMAX method [16], an evolution version of the Least Squares Complex Frequency domain (LSCF) [17] method.

In this paper, we present a new friendly-user software application for OMA based on a multivariate autoregressive model [18]. The method uses a large number of sensors which are simultaneously acquired for modal shape identification and model parameters are estimated by least squares via the computation of the QR factorization. A new criterion for the optimal order computation is developed, and the uncertainties of the identified modal parameters are computed. The software automatically computes the frequency components and coherences from the noise free spectrum.

2. VECTOR AUTOREGRESSIVE MODEL FOR OPERATIONAL MODAL ANALYSIS

By using d sensors, a pth order multivariate autoregressive model can be expressed as follows:

$$\{y(t)\}+[a_1]\{y(t-1)\}+[a_2]\{y(t-2)\}+...+[a_p]\{y(t-p)\}=\{e(t)\} \tag{1}$$

Or $$\{y(t)\}_{d\times 1}=[A]_{d\times dp}\{\varphi(t)\}_{dp\times 1}+\{e(t)\}_{d\times 1} \tag{2}$$

where $[A]_{d\times dp}=\left[-[a_1]\quad -[a_2]\quad ...\quad -[a_i]\quad ...\quad -[a_p]\right]$ is the model parameter matrix,

$[a_i]_{d\times d}$ is the matrix of autoregressive parameters relating the output $\{y(t-i)\}$ to $\{y(t)\}$,

$\{\varphi(t)\}_{dp\times 1}=\left[\{y(t-1)\};\quad \{y(t-2)\};\quad ...;\quad \{y(t-p)\}\right]$ is the regressor for the output vector $\{y(t)\}$

$\{y(t-i)\}_{d\times 1}$ (i=1:p) is the output vector with delay time $i\times T$,

T is the sampling period (s), and

$\{e(t)\}_{d\times 1}$ is the residual vector of all output channels, and is considered as the error of the model.

The data is assumed to be measured in a white noise environment that excites the natural frequencies. Fig 1 shows an example of 6 time series that were simultaneously acquired.

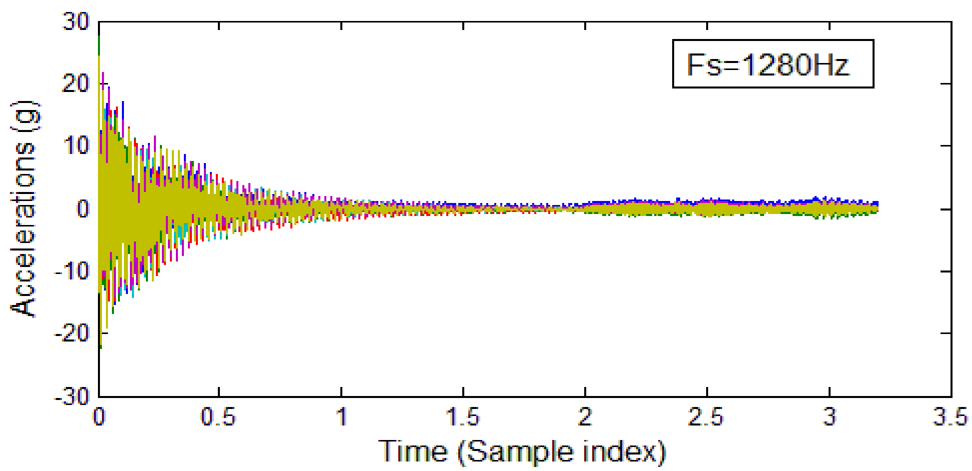

Fig. 1 Time signals

If N (N ≥ dp+d) consecutive output vectors of the responses from $\{y(k)\}$ to $\{y(k+N-1)\}$ are taken into account, the model parameters can obviously be estimated with the least squares method by minimizing a norm of error sequences.

The data matrix is first constructed from available data:

$$[K]_{N\times(dp+d)} = \begin{bmatrix} \{\varphi(t)\}_{dp}^{T} & \{y(t)\}_{d}^{T} \\ \{\varphi(t+1)\}_{dp}^{T} & \{y(t+1)\}_{d}^{T} \\ \dots & \dots \\ \{\varphi(t+N-1)\}_{dp}^{T} & \{y(t+N-1)\}_{d}^{T} \end{bmatrix} \tag{3}$$

The QR factorization [19] of the data matrix $[K]_{N\times(dp+p)} = [Q]_{N\times N}[R]_{N\times(dp+p)}$ can be computed by the Householder method or Givens rotation [20]. It gives $[Q]_{N\times N}$ which is an orthogonal matrix ($Q.Q^T = I$) and $[R]_{Nx(dp+d)}$ which is an upper triangular matrix with the form:

$$[R]_{N\times(dp+d)} = \begin{bmatrix} [R_{11}]_{dp\times dp} & [R_{12}]_{dp\times d} \\ 0 & [R_{22}]_{dxd} \\ 0 & 0 \end{bmatrix} \tag{4}$$

The model parameters matrix $[A]_{d\times dp}$ can be expressed as follows:

$$[A] = ([R_{12}^T][R_{11}]).([R_{11}^T][R_{11}])^{-1} = ([R_{11}^{-1}][R_{12}])^T \tag{5}$$

Once the model parameters are estimated, the state matrix of the system can be established in the form of autoregressive parameters:

$$[\Phi]_{dp\times dp} = \begin{bmatrix} -[a_1]_{d\times d} & -[a_2]_{d\times d} & -[a_i]_{d\times d} & \dots & -[a_p]_{d\times d} \\ I & 0 & 0 & \dots & 0 \\ 0 & I & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & I & 0 \end{bmatrix} \tag{6}$$

where the poles of the model are also the roots of the characteristic polynomial of the state matrix.

Therefore, the eigen-decomposition of the state matrix can be obtained:

$$[S|_{dp\times dp}, u|_{1\times dp}] = eig(\Phi) \tag{7}$$

The eigenvalues, frequencies, damping rates and mode shapes of the system can be computed as follows:

Eigenvalues: $\lambda_k = \dfrac{\ln(u_k)}{T_s}$

Modal vector: $l_k|_{d\times 1} = S(1:d,k)$

Frequencies: $\omega_k = \sqrt{\mathrm{Re}^2(\lambda_k) + \mathrm{Im}^2(\lambda_k)}$ (8)

Damping rates: $\xi_k = -\dfrac{\mathrm{Re}(\lambda_k)}{\omega_k}$

Mode shapes: $\Psi_k|_{d\times 1} = abs(S(1:d,k))$

As well as the estimated covariance matrices of the signal and noise parts:

$$\hat{D}\big|_{d\times d} = \left[R_{12}\right]^T R_{12}$$

$$\hat{E}\big|_{d\times d} = \left[R_{22}\right]^T R_{22} \tag{9}$$

3. ORDER UPDATING AND SELECTION OF OPTIMAL MODEL ORDER

It is observed that the selection of the optimal order is crucial in parametric modal-based methods. In this software, the above model is successively updated with respect to model order from a prior small order value.

The data matrix $K^{(p)}$ at order p can be rewritten as:

$$K^{(p)}{}_{N\times(dp+d)} = \begin{bmatrix} \varphi^T(k) & y^T(k) \\ \varphi^T(k+1) & y^T(k+1) \\ \dots & \dots \\ \varphi^T(k+N-1) & y^T(k+N-1) \end{bmatrix} = \begin{bmatrix} K_1^{(p)}{}_{N\times dp} & K_2{}_{N\times d} \end{bmatrix} \tag{10}$$

If the model order is updated to $p+1$, the data matrix takes the form:

$$K^{(p+1)}{}_{N\times(d(p+1)+d)} = \begin{bmatrix} K_1^{(p)}{}_{N\times dp} & K'_{N\times d} & K_2{}_{N\times d} \end{bmatrix} \tag{11}$$

where $K'\big|_{Nxd}$ are the added d columns

$$K'_{N\times d} = \begin{bmatrix} y^T(k-(p+1)) \\ y^T(k+1-(p+1)) \\ \dots \\ y^T(k+N-1-(p+1)) \end{bmatrix} \tag{12}$$

We can then compute the following matrix:

$$Q^{(p)T}K^{(p+1)} = \begin{bmatrix} Q^{(p)T}K_1^{(p)}{}_{N\times dp} & Q^{(p)T}K'_{N\times d} & Q^{(p)T}K_2{}_{N\times d} \end{bmatrix} = \begin{bmatrix} R_{11}^{(p)} & T_1 & R_{12}^{(p)} \\ 0 & T_2 & R_{22}^{(p)} \end{bmatrix} \tag{13}$$

where $T_1{}_{dp\times d}$ and $T_2{}_{(N-dp)\times d}$ are extracted from $Q^{(p)T}K' = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}$.

We must now triangularize the right term matrix in equation (22). This can be done with a set of Householder transformations or Givens rotations [20]. If we decompose only the small submatrix T_2, it easily yields:

$$T_2 = Q_T \begin{bmatrix} R_T \\ 0 \end{bmatrix} \tag{14}$$

where $R_{T\ d\times d}$ is an upper diagonal matrix and $Q_{T\ (N-dp)\times(N-dp)}$ is the product of the Householder transformations or Givens rotations.

Equation (14) then becomes:

$$Q^{(p)T} K^{(p+1)}{}_{N\times(d(p+1)+d)} = \begin{bmatrix} I_{dp\times dp} & 0 \\ 0 & Q_T \end{bmatrix} \begin{bmatrix} R_{11}^{(p)} & T_1 & R_{12}^{(p)} \\ 0 & R_T & Q_T^T \left[R_{22}^{(p)} \right] \\ 0 & 0 & \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} I_{dp\times dp} & 0 \\ 0 & Q_T^T \end{bmatrix} Q^{(p)T} K^{(p+1)} = \begin{bmatrix} R_{11}^{(p)} & T_1 & R_{12}^{(p)} \\ 0 & R_T & R_{22}^{'} \\ 0 & 0 & R_{22}^{''} \end{bmatrix} \tag{16}$$

where $R_{22\ d\times d}^{'}$ and $R_{22\ (N-dp-d)\times d}^{''}$ are obtained from multiplication $\begin{bmatrix} R_{22}^{'} \\ R_{22}^{''} \end{bmatrix} = Q_T^T R_{22}$.

It can be seen that the first $d \times p$ rows of the right hand side in equation (16) are not affected by the above transformations and the factor matrix $R^{(p+1)}$ at order $p+1$ was thus updated:

$$R_{11}^{(p+1)} = \begin{bmatrix} R_{11}^{(p)} & T_1 \\ 0 & R_T \end{bmatrix};\ R_{12}^{(p+1)} = \begin{bmatrix} R_{12}^{(p)} \\ R_{22}^{'} \end{bmatrix};\ R_{22}^{(p+1)} = R_{22}^{''} \tag{17}$$

as was the Q matrix:

$$Q^{(p+1)} = Q^{(p)} \begin{bmatrix} I_{dpxdp} & 0 \\ 0 & Q_T \end{bmatrix} \tag{18}$$

and the two covariance matrices from equation (9):

$$\hat{D}^{(p+1)} = \left[R_{12}^{(p+1)} \right]^T R_{12}^{(p+1)} = \left[R_{12}^{(p)} \right]^T R_{12}^{(p)} + \left[R_{22}^{'} \right]^T R_{22}^{'} = \hat{D}^{(p)} + \left[R_{22}^{'} \right]^T R_{22}^{'} \tag{19}$$

$$\hat{E}^{(p+1)} = \left[R_{22}^{(p+1)} \right]^T R_{22}^{(p+1)} = \left[R_{22}^{''} \right]^T \left[R_{22}^{''} \right] = \left[R_{22}^{(p)} \right]^T R_{22}^{(p)} - \left[R_{22}^{'} \right]^T \left[R_{22}^{'} \right] = \hat{E}^{(p)} - \left[R_{22}^{'} \right]^T \left[R_{22}^{'} \right]$$

Finding the optimal model order p_{min} is crucial for the optimization of time consumed and accuracy [21, 22]. From a statistical point of view, AIC [23] and MDL [24] criteria can be used to select the optimal model order. It is seen from equation (19) that as the model order increases, the norm of the deterministic covariance matrix increases, and that one of the error part decreases by the same amount. The global noise to signal ratio $NSR = \frac{\hat{E}}{\hat{D}}$ is therefore monotonically decreased with respect to the model order. The Noise-rate Order Factor (NOF) defines the change in the NSR within two successive order values:

$$NOF^{(p)} = \hat{NSR}^{(p)} - \hat{NSR}^{(p+1)} \qquad (20)$$

It is seen that the NOF is always positive, and that it falls quickly at a low order and converges to high orders, thus making it a criterion for model order selection. The optimal order p_{min} should therefore be chosen only after a significant change occurs in the NOF, and from that value on, the NOF will converge. Fig. 2 shows an example of the NOF behavior (extracted from a vibrating plate) which reveals that the considered system must have a minimum of 6 degrees of freedom.

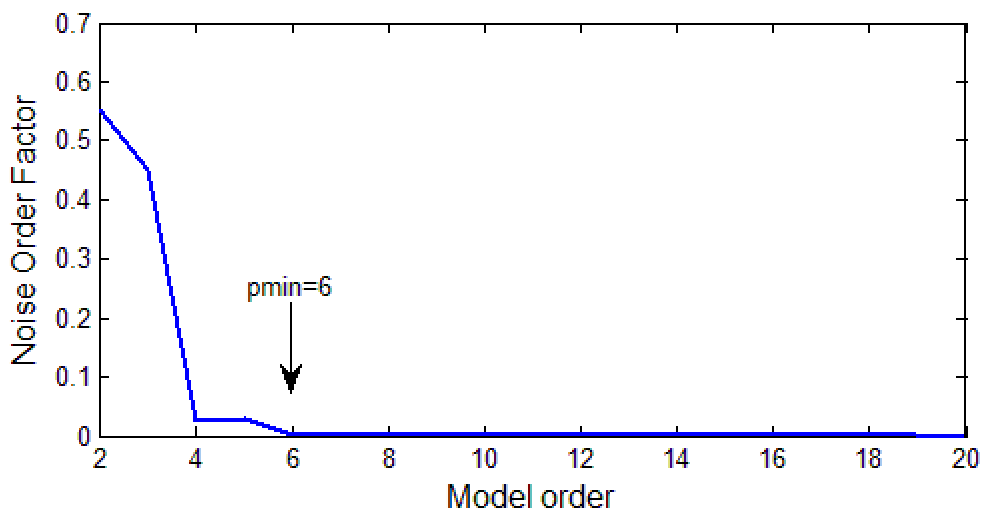

Fig. 2 NOF evolution

4. STABILIZATION DIAGRAMS OF MODAL PARAMETERS

Since the model is updated with respect to the order, using stabilization diagrams is appropriate for identifying the stable frequencies. Frequency stabilization diagrams are well known and widely used. Since the vibratory signal is noisy, the frequency stabilization diagram (Fig. 2) exhibits a lot of frequencies, containing the natural frequencies, the noise frequencies and the computational frequencies. By observing the stability of the identified frequencies with respect to increasing model order, it is possible to distinguish the physical from the spurious modes. Figure 3 shows an example (extracted from a vibrating plate) which reveals that the considered system has 13 stable natural frequencies in the 0-600 Hz range.

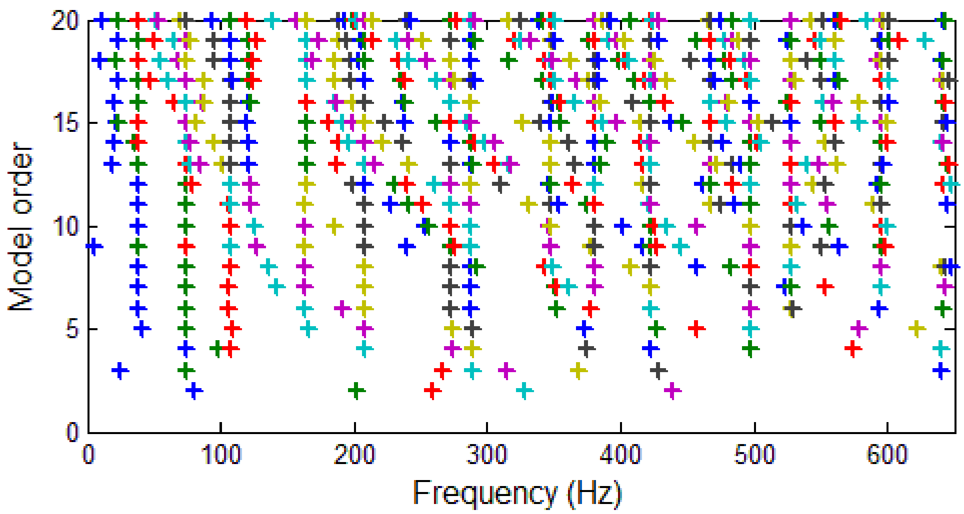

Fig. 3 Frequency stabilization diagram

Figure 4 shows a selection of stable frequencies in the 0-250 Hz range.

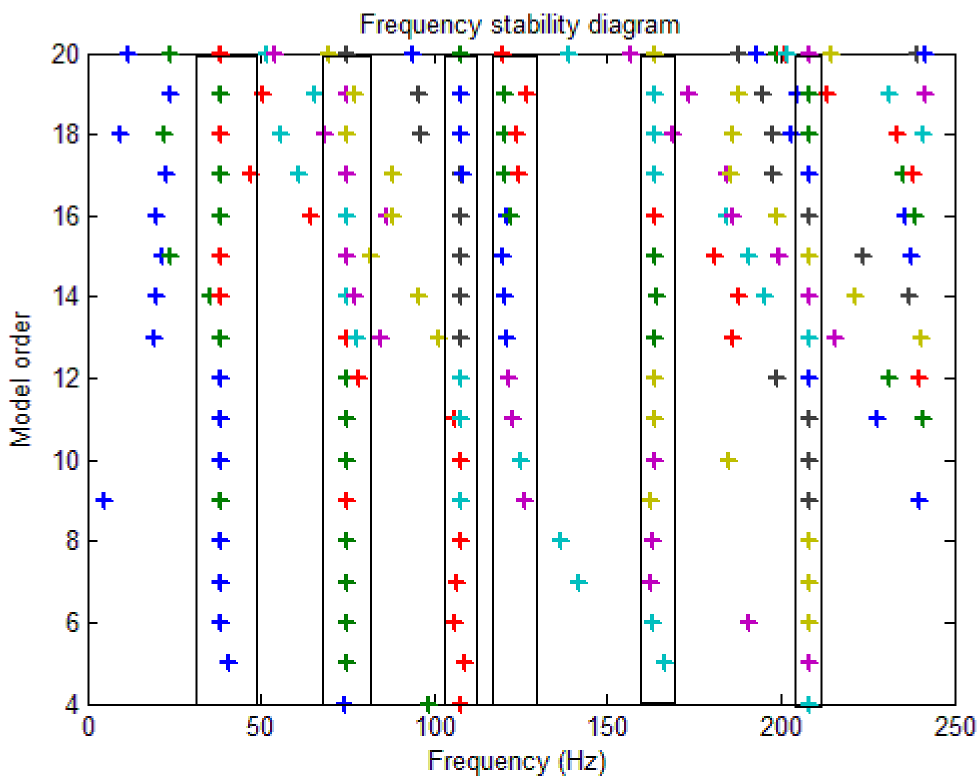

Fig. 4 Selection of stable frequencies

For each identified frequency, a stabilization diagram can be used to check the stability of the damping rate [25].

In this method, we present the stabilization for the mode shapes, which are developed by using a correlation criterion called the OMAC (Order Modal Assurance Criterion). In this new index, the correlation is constructed between the identified mode shapes [26] given by the model at order p and at its previous value $p-1$, formulated as follows:

$$OMAC_i = \frac{Cov(\Psi_i^p, \Psi_i^{p-1})}{\sqrt{Var(\Psi_i^p)}\sqrt{Var(\Psi_i^{p-1})}}. \tag{21}$$

where Ψ_i^p and Ψ_i^{p-1} are the identified real mode shape vectors at orders p and $p-1$ respectively.

Figure 5 shows an example of OMAC behavior for the first four modes.

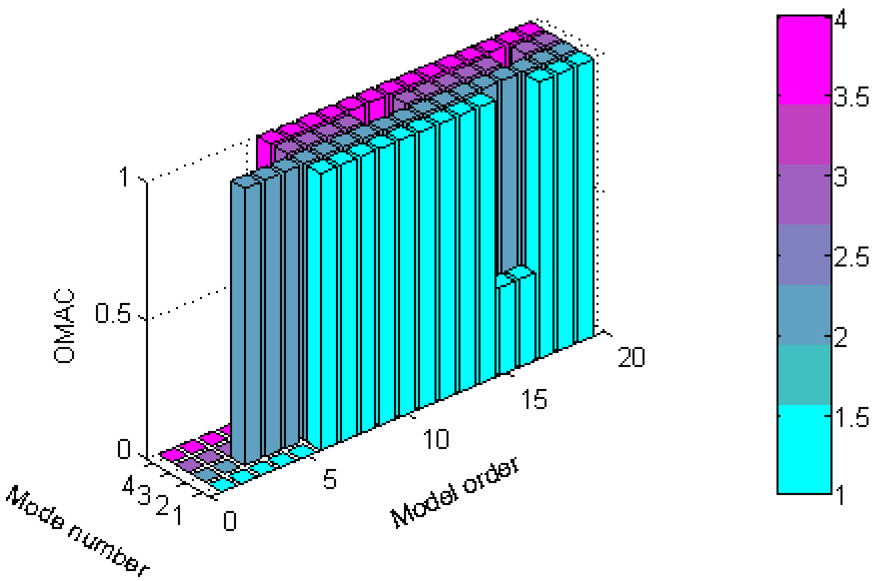

Fig. 5 Modal assurance criterion of first 4 modes

Once the modes selected, they can be plotted. Fig. 6 shows the first structural mode**.**

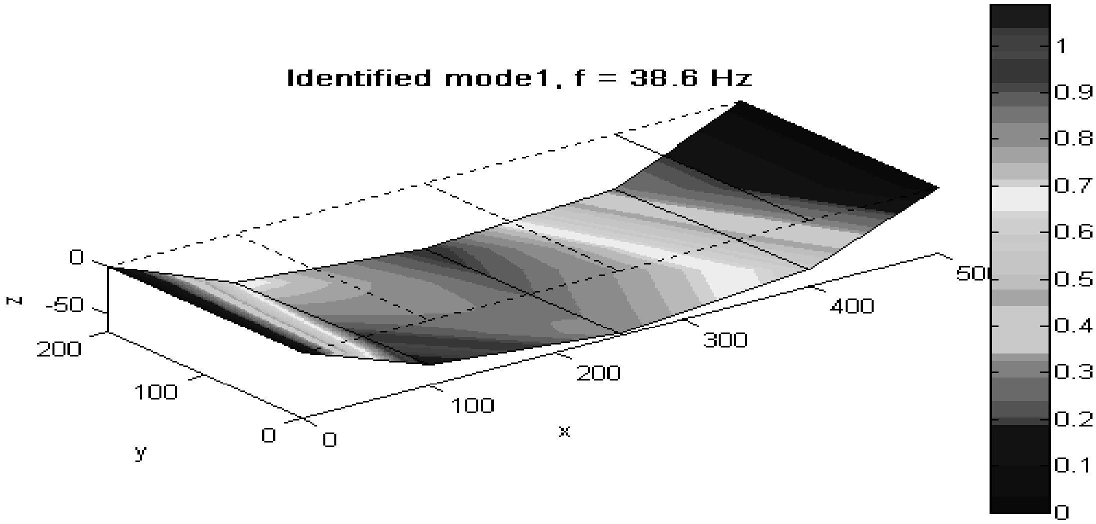

Figure 6 First mode

5. UNCERTAINTIES OF MODAL PARAMETERS

It is evident that the measurements of vibratory responses while unobserved perturbations, produces an uncertainty in the parameters estimation, and hence in the modal parameters. Therefore, an analysis of the confidence intervals of modal parameters should be taken into account [27]. Since the algorithm is updated with respect to model order, the explicit confidence intervals of the modal parameters need to be developed, and a discussion carried out on their variation with respect to model orders and noise rates.

Consider a real-valued function on the model parameters $h = h(\Phi)$. This function may consider a model parameter, a frequency, a damping rate or a mode component. The confidence interval of function $h(\Phi)$ can be constructed from the distribution of t-ratio $t = \frac{h_{\pm}}{\hat{\sigma}_h}$ where the estimated error is $h_{\pm} = \hat{h} - h$ with the estimated variance $\hat{\sigma}_h^2 = Cov(h(\hat{\Phi}))$ and $N - d^2 p$ degrees of freedom [19]. It means that its $100\,\alpha$ % confidence has the error margin:

$$\hat{h}_{\pm} = t(N - d^2 p, (1+\alpha)/2)\hat{\sigma}_h \tag{22}$$

The covariance of the estimated function $Cov(h(\hat{\Phi}))$ can be derived from its linearization at the first term of Taylor series and is guaranteed to be positive (semi-) definite [24].

$$Cov(h(\hat{\Phi})) = (\frac{\partial h(\Phi)}{\partial \Phi_0})^T Cov(\hat{\Phi})(\frac{\partial h(\Phi)}{\partial \Phi_0}) \tag{23}$$

where the covariance matrix of the least squares estimator $Cov(\hat{\Phi})\Big|_{d^2p\times d^2p}$ is the Kronecker product of the noise covariance matrix and the moment matrix $U\Big|_{dp\times dp}$ and can be derived as follows:

$$Cov(\hat{\Phi})\Big|_{d^2p\times d^2p} = U^{-1}\Big|_{dp\times dp} \otimes \hat{E}\Big|_{d\times d} = (R_1^T R_1)^{-1} \otimes (R_3^T R_3) \tag{24}$$

With the t-distribution assumption, the construction of confidence intervals of any function is based on the computation of its derivative $\frac{\partial h(\Phi)}{\partial \Phi_0}$ with respect to model parameters.

It has been shown in [27] that the derivative of the k^{th} discrete eigenvalue of the state matrix is:

$$\dot{u}_k = (S^{-1}\dot{A}S)_{kk} \tag{25}$$

Thus, the derivative of continuous eigenvalues can be derived from equation (25):

$$\dot{\lambda}_k = \frac{\dot{u}_k}{u_k T_s} = \frac{(S^{-1}\dot{A}S)_{kk}}{u_k T_s} \tag{26}$$

The natural frequencies, damping ratios and their derivatives are calculated in a straightforward manner:

$$f_k = \frac{|\lambda_k|}{2\pi} = \frac{\sqrt{\mathrm{Re}^2\,\lambda_k + \mathrm{Im}^2\,\lambda_k}}{2\pi};\;\; \dot{f}_k = \frac{\mathrm{Re}\,\lambda_k\,\mathrm{Re}\,\dot{\lambda}_k + \mathrm{Im}\,\lambda_k\,\mathrm{Im}\,\dot{\lambda}_k}{4\pi^2 f_k} \tag{27}$$

$$\zeta_k = -\frac{\mathrm{Re}\,\lambda_k}{|\lambda_k|};\; \dot{\zeta}_k = \zeta_k\left(\frac{\mathrm{Re}\,\dot{\lambda}_k}{\mathrm{Re}\,\lambda_k} + \frac{\dot{f}_k}{f_k}\right). \tag{28}$$

The real mode shapes are taken from the amplitude of complex eigenvectors and thus the partial derivative of a component of mode shapes component is ultimately obtained as follows:

$$\Psi_k(i)^2 = \mathrm{Re}^2(S_{i,k}) + \mathrm{Im}^2(S_{i,k}) \qquad \mathrm{i} = 1:d \tag{29}$$

$$\Psi_k(i)\dot{\Psi}_k(i) = \mathrm{Re}(S_{i,k})\,\mathrm{Re}(\dot{S}_{i,k}) + \mathrm{Im}(S_{i,k})\,\mathrm{Im}(\dot{S}_{i,k}) \tag{30}$$

where the complex partial derivative $\dot{S}$ was taken from the derivation of the eigen-decomposition and of the normalization of the modes shapes, as given in [27].

Figure 7 shows an example of stabilization diagram of the modal parameter uncertainties with a confidence interval of 95%. This type of result helps the analyst to validate, select and make a decision with respect to the accuracy of modal parameter estimation.

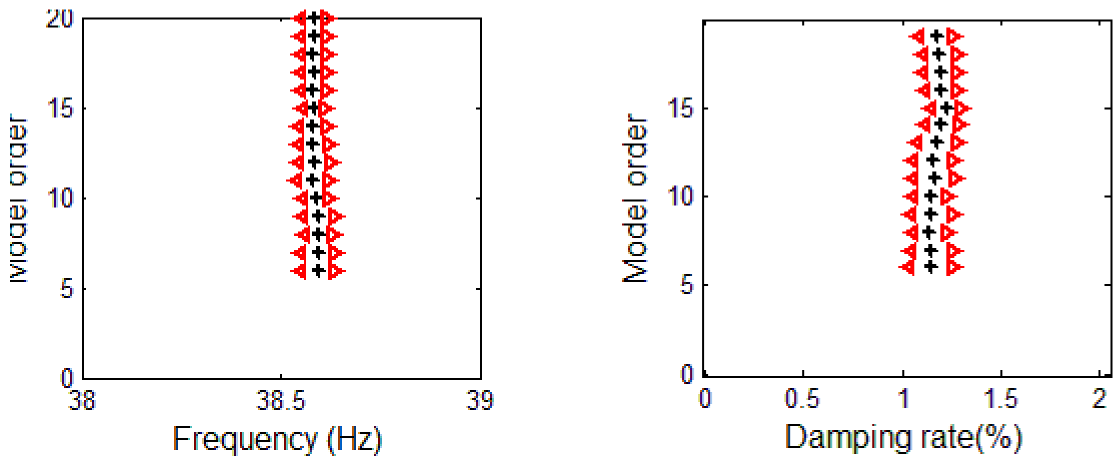

Fig. 7 Stabilization diagram of modal parameter uncertainties with 95% confidence interval

6. CONSTRUCTION OF NOISE FREE SPECTRA

It is evident that the construction of a noise-free spectrum is of great interest in modal analysis. Suppose that $2n$ modes in conjugate pairs (eigenvalues and eigenvectors) are selected from the deterministic part: the spectral matrix function can be computed directly from the eigen-decomposition as follows [29]:

$$P(\omega) = \frac{T_s}{2\pi}\left[\sum_{i=1}^{2n}[v_i](1-u_i^2)\left|1-u_i e^{-j\omega T_s}\right|^2\right] \tag{31}$$

where

$$[v_i]_{d\times d} = \sum_{k=1}^{2n}\frac{[g_i]\left[\hat{E}\right][g_k]^H}{(1+u_i u_k)} \quad \text{and} \quad [g_i]_{d\times d} = \{L_{str}(1:d,i)\}\times\left[L_{str}{}^{-1}(i,1:d)\right] \tag{32}$$

Since only physical modes are taken into account, it is clear that the spectrum is decomposed into the sum of the individual frequency spectra which exhibits the noise-free peaks. These frequencies correspond to the frequencies from the deterministic parts which include natural and excitation frequencies (if present). Figure 8 shows an example of frequency identification from the noise-free power spectrum density (PSD), averaged across six sensors.

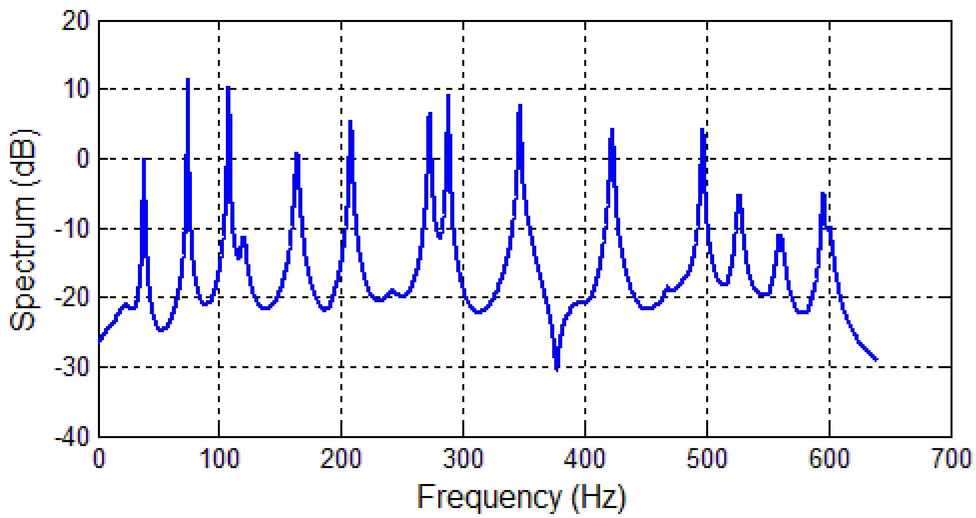

Fig. 8 Averaged power spectral density

Further calculations on the multichannel spectral matrix such as those involving channels coherence function and phase may prove interesting in [28]. Figure 9 shows that the coherence helps in selecting the most significant frequencies (close to 1).

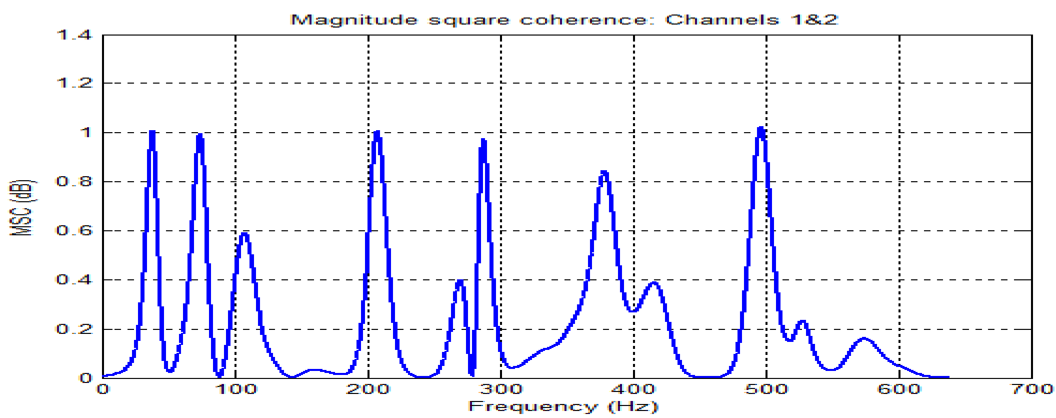

Figure 9 Coherence

7. CONCLUSION

A user-friendly software application for Operational Modal Analysis OMA (built into Matlab) is presented, based on the modeling of a multivariate autoregressive model. OMA only needs multi-channel accelerations, and does not require the measurement of excitation forces. The method uses a large number of sensors which are simultaneously acquired for modal shape identification and allow the model's parameters to be estimated. After the optimal order is selected, the uncertainties of the modal parameters identified are automatically derived in order to help the analyst to validate the accuracy of the identification. Mode shapes are classified, and a new OMAC index allows for their selection. Furthermore, the frequency components and coherences are computed from the noise-free spectrum in order to select the frequencies of interest.

8. ACKNOWLEDGEMENTS

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