

# Meta-Regression based Pool Size Prediction Scheme for Dynamic Selection of Classifiers

Anandarup Roy<sup>†</sup>, Rafael M.O. Cruz<sup>†</sup>, Robert Sabourin<sup>†</sup> and George D. C. Cavalcanti<sup>\*</sup>

<sup>†</sup>École de technologie supérieure, Quebec, Canada

<sup>\*</sup>Universidade Federal de Pernambuco, PE, Brazil

*roy.anandarup@gmail.com* (Anandarup Roy)

*rafaelmenelau@gmail.com* (Rafael M.O. Cruz)

*Robert.Sabourin@etsmtl.ca* (Robert Sabourin)

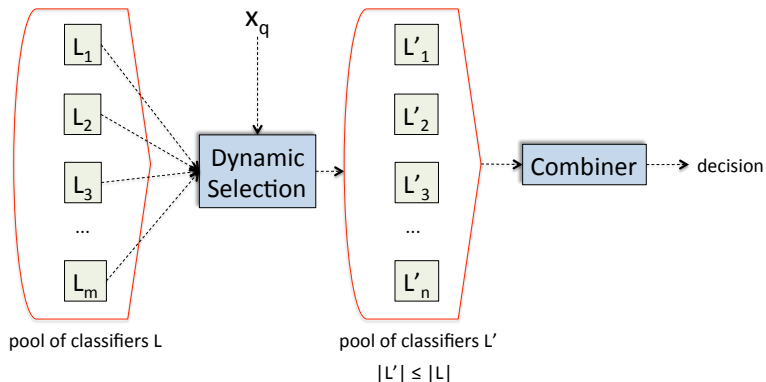
*gdcc@cin.ufpe.br* (George D. C. Cavalcanti)

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# Introduction

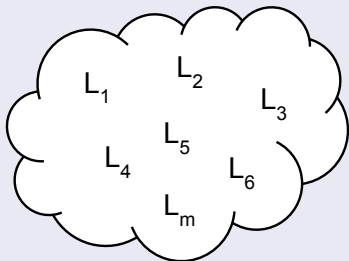
## Dynamic Selection (DS): Overview



### Variants: DCS and DES

- $|L'| = 1 \Rightarrow$  Dynamic Classifier Selection (DCS)
- $|L'| > 1 \Rightarrow$  Dynamic Ensemble Selection (DES)

## The pool of classifiers



Size of this pool ( $m$ ) is user defined yet crucial to control the computational complexity and performance of a DS.

**Objective:** How to find  $m$  ?

## Our Approach

- A meta-regression model to **predict** pool size  $m$ .
- Our method is based on the concept of **meta-learning**.
- We emphasize **classification complexity** of a problem.
- Use regression to associate classification complexity with pool size.

## Taxonomy of the complexity measures

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### Measures of overlap in feature from different classes

- F1* Maximum Fishers discriminant ratio
- F1<sub>v</sub>* Directional-vector maximum Fishers discriminant ratio
- F2* Overlap of the per-class bounding boxes
- F3* Maximum individual feature efficiency
- F4* Collective feature efficiency

### Measures of separability of classes

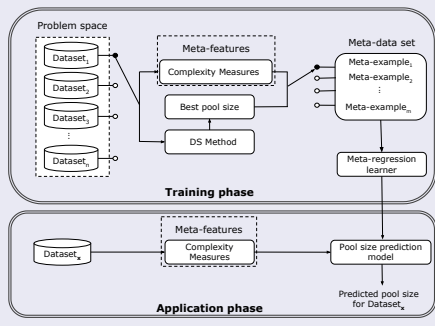
- L1* Minimized sum of error distance of a linear classifier
- L2* Training error of a linear classifier
- N1* Fraction of points on the class boundary
- N2* Ratio of average intra/inter class 1-NN distance
- N3* Leave-one-out error rate of the 1-NN

### Measures of geometry, topology and density of manifolds

- L3* Non-linearity of a linear classifier
  - N4* Non-linearity of the one-nearest neighbor classifier
  - T1* Fraction of maximum covering spheres
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Ho and Basu, Complexity Measures of Supervised Classification Problems, IEEE PAMI, 24 (3), 2002.

## Framework for pool size prediction.



**Advantage:** No need to employ DS method to know pool size.

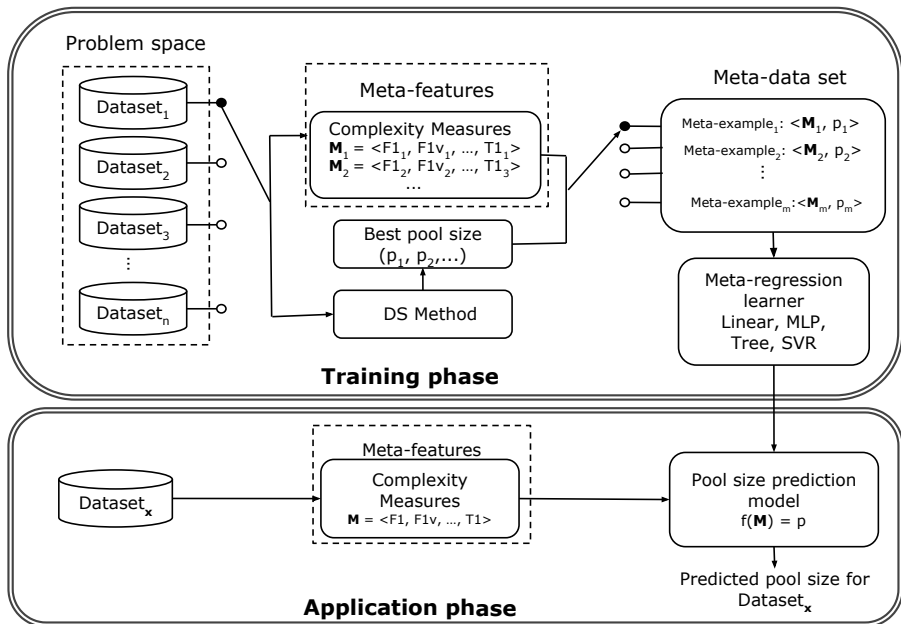
## Training Phase:

- Gather large number of data sets.
- Obtain meta-features i.e., complexity measures.
- Obtain *best pool size*: the pool size with highest accuracy of DS.
- Combine to form meta-examples.
- Find a **meta-regression model**.

## Application phase:

- Obtain meta-features for test data set
- **Predict** pool size using trained meta-regression model.

# Meta-Regression Framework



# Experimental Protocol

## Data Sets: Overview

Key Features of the data sets used in the experiments

Data set	# Inst.	# Attr.	# class	Source
Adult	48842	14	2	UCI
Banana	1000	2	2	Synthetic
Blood transfusion	748	4	2	UCI
Breast (WDBC)	568	30	2	UCI
CTG	2126	21	3	UCI
Ecoli	336	7	6	UCI
German credit	1000	20	2	STATLOG
Glass	214	9	6	UCI
Haberman	306	3	2	UCI
Heart	270	13	2	STATLOG
ILPD	214	10	2	UCI
Ionosphere	315	34	2	UCI
Laryngeal1	213	16	2	LKC
Laryngeal3	353	16	3	LKC
Lithuanian	1000	2	2	Synthetic
Liver disorders	345	6	2	UCI
Magic Gamma	19020	10	2	KEEL
Mammographic	961	5	2	KEEL
Monk2	4322	6	2	KEEL
Phoneme	5404	6	2	ELENA
Pima	768	8	2	UCI
Satimage	6435	19	6	STATLOG
Sonar	208	60	2	UCI
Steel plate faults	1941	27	7	UCI
Thyroid	215	5	3	LKC
Vehicle	846	18	4	STATLOG
Vertebral column	310	6	2	UCI
WDG V1	50000	21	3	UCI
Weaning	302	17	2	LKC
Wine	178	13	3	UCI

- Repositories: UCI, STATLOG project, KEEL and Ludmila Kuncheva medical data. Two data sets are artificial.
- Multi-class data sets are divided using one-versus-all strategy to generate two-class data sets.
- Combining, we have **64** two-class data sets.
- Each data set is partitioned into 10 subsets for 10 fold CV.
- 20 pools are generated for each subset. Pools have 5, 10, 15, ..., 150 perceptron classifiers.



### Dynamic Classifier Selection (DCS)

- **Rank:** Classifier ranking
  - Sabourin et al., 1993
- **LCA:** Local Classifier Accuracy
  - Woods et al., 1997
- **MCB:** Multiple Classifier Behaviour
  - Giacinto, 2001

### Dynamic Ensemble Selection (DES)

- **PRC:** Probabilistic Reference Classifier
  - Woloszynski 2010
- **KNORA-U:** K-Nearests Oracles-Union
  - Ko et al., Pattern Recognition 2008
- **KNORA-E:** K-Nearests Oracles-Eliminate
  - Ko et al., Pattern Recognition 2008

### Linear and Non-Parametric Models

- Linear regression without interaction terms (Linear).
- Multivariate adaptive regression splines (MARS) without interaction (MARS-0) and with piecewise interaction (MARS-1)

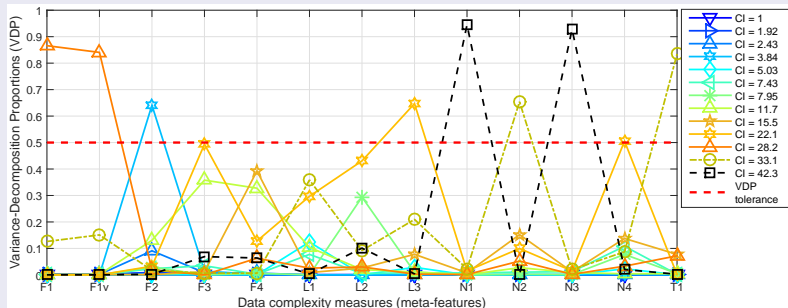
### Tree and Non-Linear Models

- Decision tree with binary split (Tree)
- Multi-layer perceptron (MLP) with 6 ( $MLP_6$ ) and 8 ( $MLP_8$ ) hidden nodes.
- Support vector regression (SVR) with linear (SVR-L) and Gaussian (SVR-G) kernels.

# Experimental Results

## Multicollinearity Detection for meta-features

### Belsley collinearity diagnostics



- **Multicollinearity:** two or more predictor variables are highly linearly related.
- The tolerances for CI and VDP are 35 and 0.5.
- A wide gap between last two CI values.
- *N1* and *N3* exhibit collinearity.
- **Remedy:** Remove *N3* from meta-features.

# Experimental Results

## Meta-regression predictive performance

**Squared prediction Errors (SpE):** Error between “average predicted” and the “average best” pool sizes. We perform paired t-test to compare the SpE of 64 data sets for each pair of algorithms.

Pairwise comparison between meta-regression algorithms.

	DS methods	Meta-regression algorithms							
		Linear	Tree	$MLP_6$	$MLP_8$	SVR-L	SVR-G	MARS-0	MARS-1
DES	PRC	(1,5,1)	(0,2,5)	(5,2,0)	(4,3,0)	(1,4,2)	(0,7,0)	(0,5,2)	(1,4,2)
	KNORA-U	(1,4,2)	(1,4,2)	(6,1,0)	(6,1,0)	(1,4,2)	(0,0,7)	(1,4,2)	(1,4,2)
	KNORA-E	(1,4,2)	(0,1,6)	(5,2,0)	(6,1,0)	(1,4,2)	(0,6,1)	(1,4,2)	(1,4,2)
DCS	Rank	(0,7,0)	(0,5,2)	(2,5,0)	(3,4,0)	(0,6,1)	(0,7,0)	(0,5,2)	(0,7,0)
	LCA	(2,4,1)	(0,2,5)	(4,3,0)	(6,1,0)	(1,4,2)	(0,1,6)	(2,4,1)	(2,3,2)
	MCB	(1,6,0)	(0,4,3)	(3,4,0)	(3,4,0)	(0,7,0)	(0,7,0)	(0,5,2)	(0,5,2)

- Each cell presents amount of win, tie and loss for an algorithm over all its competitors.

### Observations

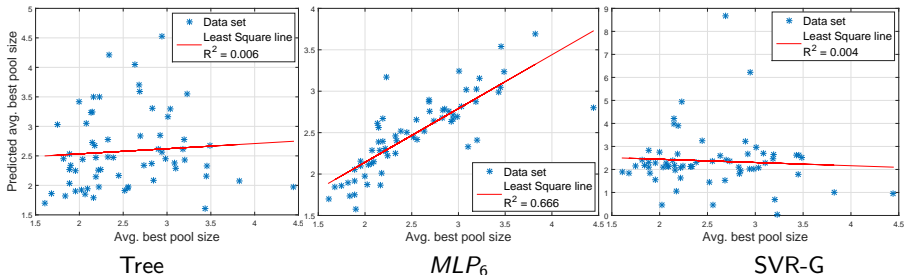
- SVR-G never outperform any of its competitors.
- Tree, except one case, never outperform any of its competitors.
- Inferior prediction performance of Tree and SVR-G.
- $MLP_6$  and  $MLP_8$  outperform their competitors in many cases.

# Experimental Results

## Meta-regression performance comparison

Predict the average pool size: Performance evaluation

Illustration for PRC method

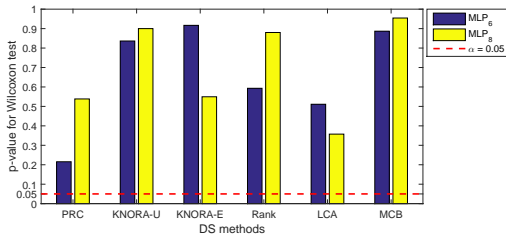


### Observations

- Predictions using Tree are more scattered ( $\rho = 0.08$ ) compare to  $MLP_6$  ( $\rho = 0.82$ ).
- Predictions yield by SVR-G are also scattered ( $\rho = -0.07$ ).
- For SVR-G: Negative slope of the regression line  $\Rightarrow$  unreliable predictions.
- $MLP_6$  provides predictions close to the actual.

# Experimental Results

Generalization performance of DS for predicted pool size



Wilcoxon signed rank test to assess equivalence between classification accuracies between DS with best pool size ( $DS_b$ ) and DS with predicted pool size ( $DS_p$ ).

## Observations

- For both  $MLP_6$  and  $MLP_8$ ,  $DS_p$  performs equivalent to  $DS_b$ .
- The accuracy in prediction is reflected in the DS performance.
- DS methods are robust enough to withstand certain prediction errors.

## Conclusions

- Meta-regression (specially MLP) can be an useful tool for predicting the pool size of a DS.
- Compare to brute force, our technique is cost effective.
- Complexity measures are competent meta-features for pool size prediction.

## Future Scope

- More data sets to cover whole complexity space.
- Include other categories of meta-features for multi-class problems.
- Cluster the data sets according to meta-features to find more accurate regression models.
- Ensemble of regression models for a better prediction.

THANK YOU