# A 3-$\underline{R}PR$ Parallel Mechanism With Singularities That are Self-Motions

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*The Cartesian workspace of most three-degree-of-freedom parallel mechanisms is divided by Type 2 (also called parallel) singularity surfaces into several regions. Accessing more than one such region requires crossing a Type 2 singularity, which is risky and calls for sophisticated control strategies. Some mechanisms can still cross these Type 2 singularity surfaces through "holes" that represent Type 1 (also called serial) singularities only. However, what is even more desirable is if these Type 2 singularity surfaces were curves instead. Indeed, there exists at least one such parallel mechanism (the agile eye) and all of its singularities are self-motions. This paper presents another parallel mechanism, a planar one, whose singularities are self-motions. The singularities of this novel mechanism are studied in detail. While the Type 2 singularities in the Cartesian space still constitute a surface, they degenerate into lines in the active-joint space, which is the main result of this paper.* [DOI: 10.1115/1.4001737]

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## 1 Introduction

[1]Most parallel robots have singularities that limit their workspace. The less harmful ones occur when one leg is at a singularity or, in other words, when the mobile platform looses one or more degrees of freedom (DOF). We loosely call these *Type 1 singularities* (other authors call them *serial singularities*). The most dangerous singularities, although, are those associated with the loss of stiffness of the mobile platform, which we loosely call *Type 2 singularities* (sometimes also referred to as *parallel singularities*). Indeed, approaching a Type 2 singularity often requires large actuator torques or forces. Hence, these singularities should be avoided for most applications. A safe solution is to eliminate Type 2 singularities at the very design stage through optimization [1,2] or to determine singularity-free zones in the workspace [3] and limit the motion of the mobile platform through careful trajectory planning. In industry, although, the first approach seems to be the only one adopted to date.

For an $n$-DOF parallel mechanism, the Type 2 singularity loci in the Cartesian space generally constitute an $n-1$ dimensional entity. For example, the singularity loci of a general 3-DOF 3-$\underline{R}RR$ spherical parallel mechanism constitute a two-dimensional surface separating the 3D (orientation) workspace into *aspects*. In Ref. [4], however, it was demonstrated that for a 3-$\underline{R}RR$ parallel wrist in which all mutually fixed axes are normal, a design known as the *Agile Eye*, Type 2 singularity surfaces degenerate into six curves. What is more, these curves correspond to both Type 1 and Type 2 singularities as well as to *self-motions*. Self-motions, are certainly the worst kind of singularity and few mechanisms have such singularities but, then, if the singularity surface is reduced to curves, the complete 3D workspace (excluding these curves) becomes attainable and this is certainly worth the cost. After all, we simply need to avoid self-motions in the same way that we avoid an "ordinary" Type 2 singularity.

This led the second author to look for other such parallel mechanisms with degenerate singularities of reduced dimension under the hypothesis that this degeneracy occurs when Type 1 and Type 2 singularities coincide and the latter are self-motions. In the case of 3-DOF planar parallel mechanisms, the best "equivalent" of the agile eye seems to be the 3-$\underline{R}PR$ mechanism, which is largely overlooked in literature. Both a particular design and the general design were studied [5,6] and while they had some interesting (Cardanic) self-motions, no design with degenerate singularity loci was found.

Meanwhile, it was demonstrated in Ref. [7] that a 3-$R\underline{P}R$ planar parallel mechanism can cross the Type 2 singularity loci at special "holes" generally representing Type 1 singularities only. Passing through these holes is not always easy to do in practice but, in theory, the mechanism is not necessarily even close to Type 2 singularities when this passage occurs, whatever this measure of closeness is (e.g., Ref. [8]). Thus, the singularity loci of these mechanisms do not divide the Cartesian workspace into two parts. Such holes in the Type 2 singularity loci also exist in the case of 3-$\underline{R}PR$ planar parallel mechanisms.

The results presented in this paper bring us one step closer to finding a planar equivalent of the agile eye in terms of reduced singularity manifolds. Specifically, we present a special 3-$\underline{R}PR$ design in which the Type 2 singularity loci in the $xy\phi$ Cartesian space are the same for any orientation—a circle—which is rarely the case for any 3-DOF planar parallel robot. Furthermore, all Type 2 singularities are actually (Cardanic) self-motions and all Type 1 singularities belong to the circle of Type 2 singularities. Finally, while Type 2 singularities in the $xy\phi$ Cartesian space still constitute a surface (a cylinder), they degenerate into (three) lines in the active-joint space.

## 2 Singularities of 3-$\underline{R}PR$ Planar Parallel Mechanisms With Coincident Base Joints

Figure 1 shows a general 3-$\underline{R}PR$ planar parallel mechanism. We denote with $O_i$ and $B_i$ (in this paper, $i=1,2,3$) the intersections of the base and platform revolute joint axes, respectively, with a plane normal to these axes. Then, let $Oxy$ and $Cx'y'$ be the base and mobile reference frames, respectively, (not shown). We define each active-joint variable $\theta_i$ as the angle between the $x$ axis and the direction of the prismatic joint of leg $i$, measured in the counterclockwise sense. Finally, the distance from point $O_i$ to point $B_i$ along the positive direction of prismatic joint $i$ is denoted by $\rho_i$.

Let the axes of all active joints coincide and pass through point $O$, as shown in Fig. 2, in order to maximize the workspace of the mechanism. Even in practice, such a mechanism can rotate to any orientation. Reference [6] presented, in detail, the equations for the direct and inverse kinematic problems of a general 3-$\underline{R}PR$ planar parallel mechanism so we will not reproduce them here. The direct kinematics will be slightly simpler in our case and the two possible *assembly modes*, as shown in Fig. 2. The two solutions to the direct kinematics are diametrically opposite with respect to $O$ and the signs of the corresponding $\rho_i$ are also opposite. What is more interesting is the singularity analysis of this particular design.

The reciprocal screw for an $\underline{R}PR$ leg passes through point $B_i$ and is normal to the direction of prismatic joint $i$, as shown in Fig. 3 [9]. A Type 2 singularity occurs when the mobile platform obtains additional degrees of freedom, i.e., when the three recip-

[1]It is customary to refer to parallel mechanisms using the symbols $P$ and $R$, which stand for prismatic and revolute joints, respectively. When a joint is actuated, its symbol is underlined.

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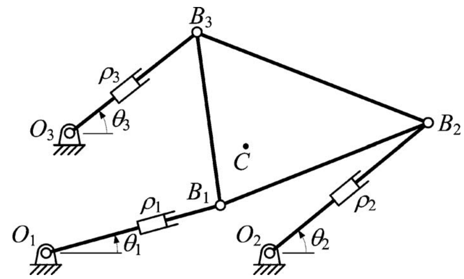


**Fig. 1 General 3-$\underline{R}PR$ planar parallel mechanism**

rocal screws intersect at one point, or are parallel to one another. The latter case is only possible when points $B_i$ are collinear and will not be studied here because it is not of interest. An elementary geometric analysis shows that if the three screws intersect at a common point $H$, then both $H$ and $O$ lie on the circumcircle of the mobile platform, the center of which is denoted by $Q$. Thus, it is obvious that a Type 2 singularity occurs if and only if $O$ lies on the circumcircle of the mobile platform but does not coincide with any of the platform vertices $B_i$ (Fig. 3(*b*)).

Therefore, the Type 2 singularity loci for a constant orientation of the mobile platform form a circle of the same radius $r$ as the circumcircle of the mobile platform but the center $P$ of which is offset from point $O$ by the vector connecting $Q$ with $C$, as shown in Fig. 4.

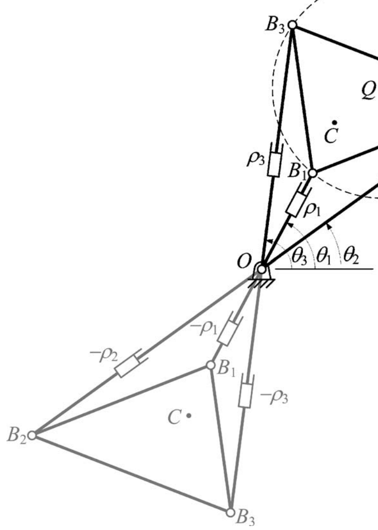


**Fig. 2 The new 3-$\underline{R}PR$ planar parallel mechanism and its two assembly modes**

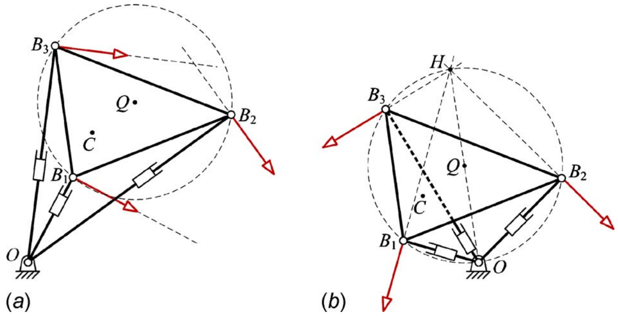


**Fig. 3 Reciprocal screws and Type 2 singularities: (*a*) a nonsingular configuration and (*b*) a Type 2 singular configuration**

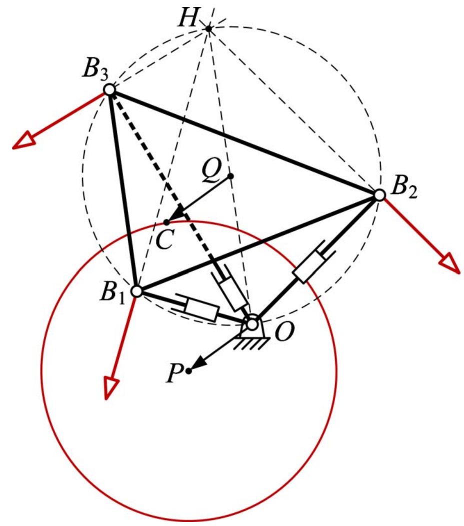


**Fig. 4 Type 2 singularity loci (the solid-line circle) for a given orientation**

It is possible to prove that in a Type 2 singular configuration, the platform can undergo finite motion even though all the actuators are fixed, as shown in Fig. 5. During this so-called self-motion, point $Q$ will describe a circle of radius $r$ (not shown) while point $C$ will describe an ellipse (shown as a dashed line). This self-motion is called Cardanic motion and is described in detail in Ref. [6]. What is highly peculiar in this design with coincident base joints is that all Type 2 singularities correspond to self-motions.

For the parallel mechanism shown in Figs. 2–5, the set of platform positions (i.e., locations of point $C$) that correspond to a singularity for at least one orientation is an annular region obtained by sweeping a circle of radius $r$ along a circle of radius $\|QC\|$ and centered at $O$. Therefore, such a choice of platform center (tooltip) is obviously not optimal. Choosing the platform center to be coincident with the center of the platform circumcircle $Q$ leads to degeneration of the annular region to a single circle of radius $r$ and centered at $O$. In other words, if $C$ is the center of the platform circumcircle, then the Type 2 singularity loci of our mechanism will constitute the same circle of radius $r$ but centered at $O$ for any orientation, i.e., a cylinder in the $xy\phi$ Cartesian space. Furthermore, as already mentioned, in the Car-

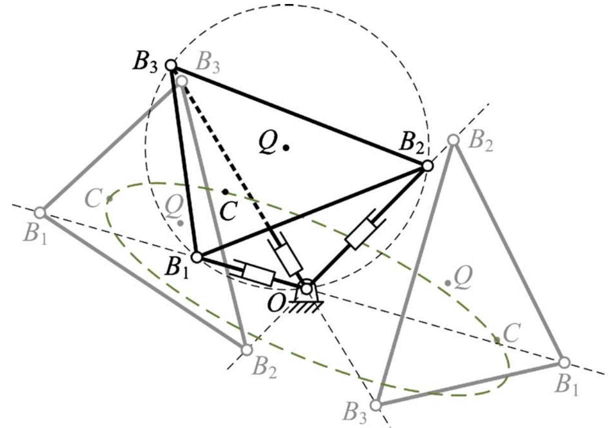


**Fig. 5 Cardanic self-motion of the mobile platform present in any Type 2 singular configuration**

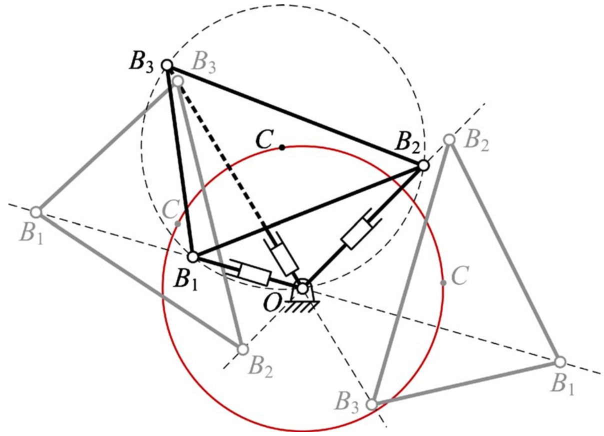


**Fig. 6 Type 2 singularity loci form the same circle for any orientation**

danic self-motion that exist in any Type 2 singular configuration, the platform center will describe the same circle of radius $r$ and center $O$, as shown in Fig. 6.

Finally, it is easy to show that the following relations between the active-joint variables generate all Type 2 singular configurations:

$$\theta_2 = \theta_1 + \beta_3, \quad \theta_3 = \theta_1 - \beta_2 + \pi \tag{1}$$

$$\theta_2 = \theta_1 + \beta_3 - \pi, \quad \theta_3 = \theta_1 - \beta_2 \tag{2}$$

$$\theta_2 = \theta_1 + \beta_3, \quad \theta_3 = \theta_1 - \beta_2 \tag{3}$$

for any $\theta_1$, where $\beta_2$ and $\beta_3$ are the platform triangle interior angles at $B_2$ and $B_3$, respectively. Therefore, in the joint space, Type 2 singularity loci correspond to three lines and not to surfaces, as is the case for all other 3-DOF parallel mechanisms that we know about.

Indeed, Fig. 7 shows Type 2 singularity loci in the active-joint

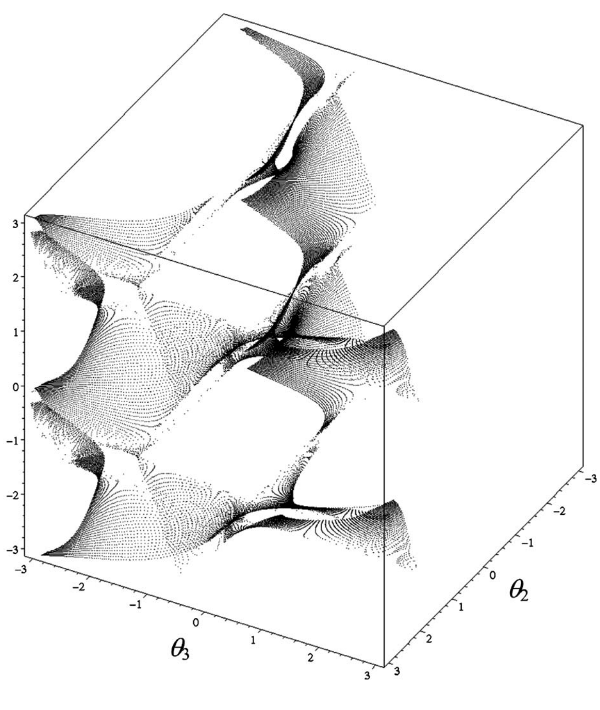


**Fig. 7 Type 2 singularity loci in the active-joint space of a general 3-$\underline{R}PR$ planar parallel mechanism**

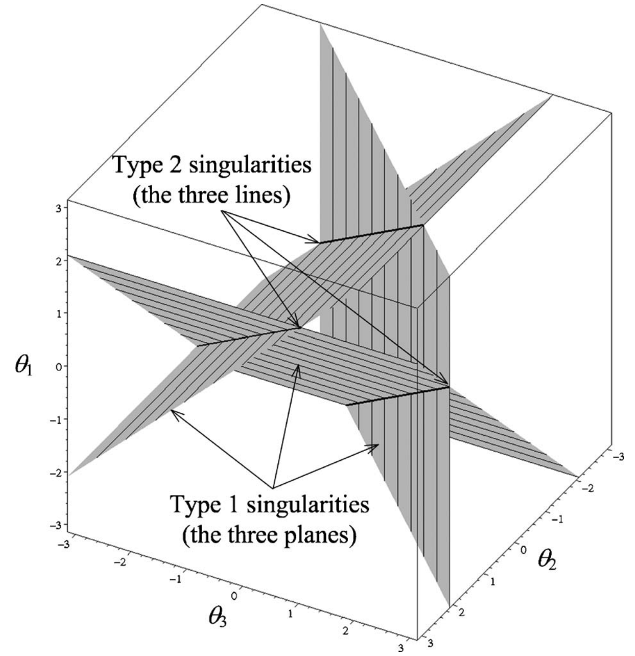


**Fig. 8 Type 1 and Type 2 singularity loci in the active-joint space for the mechanism studied**

space of a general 3-$\underline{R}PR$ planar parallel mechanism whose base joints are not coincident. Note that this figure would essentially be the same even if the base and the mobile platform are congruent equilateral triangles (i.e., the mechanism studied in Ref. [5]). These singularity loci are obviously a very complex surface.

Figure 8 shows the singularity loci in the case of the parallel mechanism under study, i.e., whose base joints are coincident. Type 2 singularity loci degenerate to three (parallel) lines. Each set of active-joint variables from one of these three lines corresponds to a 1-DOF self-motion of the mobile platform. Type 1 singularities, by contrast, correspond to planes passing through each two of these three lines in the active-joint space. Each of these planes is naturally parallel to one of the active-joint variable axes when leg $i$ is in (Type 1) singularity, $\theta_i$ can be arbitrary. Note that the gridlines of the three Type 1 singularity planes are parallel to each of the three axes. Each such line corresponds to rotating one of the motors without affecting the mobile platform.

## 3 Type 1 Singularities Providing a Passage Through Type 2 Singularities

So far, we showed that for any 3-$\underline{R}PR$ planar parallel mechanism with coincident base joints, triangular mobile platform, and a platform center coincident with the center of the platform's circumcircle, all Type 2 singularities correspond to Cardanic self-motions and their loci form a circle of the same radius as the platform's circumcircle and centered at $O$. Thus, the $xy\phi$ Cartesian workspace of this mechanism is divided into two parts that are free of Type 2 singularities. We will now see that these two parts are actually connected.

A Type 1 singularity exists when the axes of the two revolute joints in a leg coincide, as shown in Fig. 9. In this figure, the mobile platform is shown as a star rather than as a triangle for clarity. In such a configuration, the corresponding motor can undergo full-cycle motion while the mobile platform remains fixed. For only two of all possible orientations of this motor, there is actually a Type 2 singularity as well. (Recall Fig. 8 in which each grid line of one of the Type 1 singularity planes intersects two of the Type 2 singularity lines.) In other words, for a given orientation, the mobile platform can cross the Type 2 singularity circle,

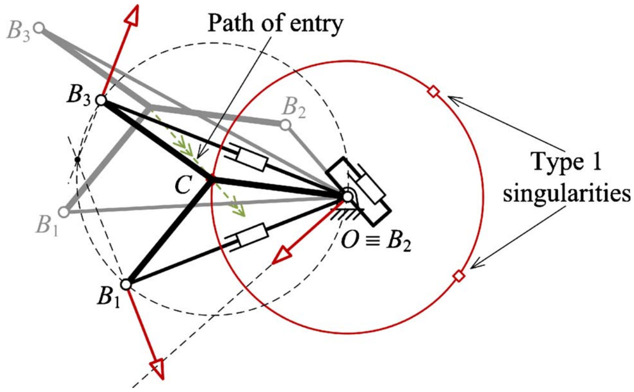


**Fig. 9 For a given orientation, three points from the Type 2 singularity circle correspond to Type 1 singularities**

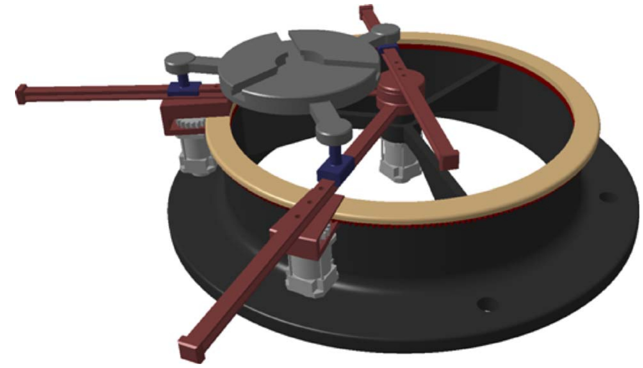

**Fig. 10 Proposed mechanical design of the mechanism under study**

without entering a Type 2 singularity, at three distinct points as long as the entry path is not tangential to the circle. In fact, a radial path in which the mobile platform, with a fixed orientation, follows a linear trajectory with point $C$ passing through $O$ and one of the three Type 1 singularity points is not only free of Type 2 singularities but far away from them (whatever the measure of closeness). Indeed, a look at Fig. 8 clearly shows that Type 2 singularities can be easily avoided.

Since the whole $xy\phi$ Cartesian workspace is connected, it might be tempting to jump to the conclusion that the mechanism under study can switch between its two assembly modes without crossing a Type 2 singularity. Indeed, in a 3-$R\underline{P}R$ planar parallel mechanism, it is possible to change an assembly mode without passing through a Type 2 singularity [7]. By contrast, in some 3-$R\underline{P}R$ planar parallel mechanisms, each singularity-free zone in the Cartesian space corresponds to a single assembly mode [10]. In our case, to change from one of the assembly modes to the other, the signs of all $\rho_i$ should change (recall Fig. 2). Yet, when the mechanism enters or exits the Type 2 singularity circle through a Type 1 singularity, the sign of a single $\rho_i$ changes. Thus, since the two direct kinematic solutions for a given set of active variables are either both outside or both inside the circle, there is no way to change the signs of all three $\rho_i$ and remain on the same side of the circle.

## 4 Practical Design

In practice, we need not (and, in fact, cannot) have each of the three legs able to enter a Type 1 singularity. Even if only one of the legs can enter a Type 1 singularity, we can still cross the Type 2 singularity circle without passing through a Type 2 singularity. Thus, we propose a mechanical design in which only leg 1 can enter a Type 1 singularity. This design is illustrated in Fig. 10.

Referring to Fig. 10, leg 1 is the one with the shortest linear guide, attached in the middle to a plate rotated directly by the motor underneath it. The other two linear guides turn about the same axis via motors that roll along a common geared circular rail, as in the *Rotopod* hexapod of Parallel Robotic Systems Corporation (Hampton, NH) [11]. In this way, the active-joint variables are completely unlimited (or almost so, given the electric cables of motors 2 and 3, which are not shown in Fig. 10).

## 5 Conclusion

In summary, the $xy\phi$ Cartesian workspace of the proposed 3-DOF 3-$\underline{R}PR$ planar parallel mechanism is theoretically unlimited. The Type 2 singularity loci of this mechanism form the same circle for any orientation, and when the mechanism is in a Type 2 singularity, it can undergo a Cardanic self-motion. For a given orientation, the Type 2 singularity circle has three points corresponding to Type 1 singularities only. Thus, the Type 2 singularity circle does not prevent the mechanism from operating both inside and outside this circle, for a given orientation. Finally, while Type 2 singularities in the $xy\phi$ Cartesian space still constitute a surface (a cylinder), they degenerate into (three) lines in the active-joint space.

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