Aerodynamic forces based on an error analytical formulation for aeroservoelasticity studies on an F/A-18 aircraft

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Abstract: Two classical methods are used in the literature to approximate the unsteady generalized forces from the frequency domain $Q(k)$ to the Laplace domain $Q(s)$ and these methods are the least squares and the minimum state (MS). In this article, a new method is presented, called the corrected minimum state (CMS), on the basis of the standard MS approximation method. This new CMS method uses an analytical form of the error as a function of Laplace variable similar to the analytical form of the aerodynamic forces calculated with the MS method. This new method is applied to an F/A-18 aircraft and it is found that the CMS method brings improvements in the approximation results in comparison with the standard MS method. It is shown that the use of the CMS method on an F/A-18 aircraft will give better results in terms of convergence speeds and precision than the MS method.

Keywords: aerodynamics, aeroelasticity, aeroservoelasticity, flutter, approximations

1 INTRODUCTION

Aeroservoelastic interaction studies regard interactions between three disciplines on an aircraft, which are unsteady aerodynamics, aeroelasticity, and servo controls. One main aspect of these studies concerns the conversion of unsteady aerodynamic forces from frequency into Laplace domain. The most known methods for this type of conversion are the least squares (LS), matrix pade (MP), and minimum state (MS), which are implemented in most of the aeroservoelastic codes.

The LS method using second-order Padé polynomials [**1**] was implemented in ADAM (Aeroservoelastic Analysis Method for Analog or Digital Systems) developed at the Air Force Wright Aeronautical Laboratories [**2**] and STARS (STructural Analysis Routines) software [**3**] developed at the Dryden Flight Research Center, NASA.

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Each term of the aerodynamic matrix was approximated by a polynomial ratio in Laplace variable by use of the MP method [**4**]. Other modifications of the MP method were suggested in references [**5**, **6**].

The capability to enforce or relax the constraints was included in the LS, MP, and MS methods [**7**]. These capabilities were abbreviated as ELS, EMP, and EMS, and they were introduced in the aeroservoelastic computer program called Interaction of Structures, Aerodynamics, and Controls. The MS approach was included in the ASTROS computer program [**8**] developed at the NASA Langley Research Center [**9**]. This method offers savings in a number of added states with little or no loss of accuracy in modelling the aerodynamic forces. However, its applicability to the unsteady aerodynamics in the transonic and hypersonic regimes remains to be established.

The MS method was implemented in ZAERO software, which uses an expedient non-linear unsteady transonic method to generate system matrices. Reduced-order techniques using proper orthogonal decomposition and MInimum STate (MIST) methods approach reduce the system to seven states, rendering an online algorithm to be operated within fractions of 1 s [10]. A reconfigurable adaptive control system for limit cycle oscillation suppression of five F/A-18 aircraft/store configurations was shown at 5.6 and 8.8 Hz.

The convergence of the MS approximation method of unsteady generalized aerodynamic forces in the equation of motion of flexible aircraft was shown by Botez and Bigras [11] using an original feature. At each iteration, an optimal compromise is chosen between the present and the last iterations.

In two articles, Cotoi and Botez [12, 13] proposed a new approach based on the Padé approximation and used order reduction methods for the last term of the approximation, which could be seen as a transfer function of a linear system. The approximation error obtained with this new method is 12–40 times lower than that obtained with the MS method for the same number of augmented states and depends on the choice made for the model reduction method. This method has the disadvantage to remain expensive in terms of computing time.

Another method based on the Padé polynomial form, which uses the Chebyshev polynomials and their orthogonality properties, is presented by Botez *et al.* [14]. This method was applied on the Aircraft Test Model (ATM) modelled in STARS, on a business aircraft at Bombardier Aerospace, and on an F/A-18 aircraft. The error calculated by this method was less than 0.814 per cent. It was found that the computation time for the Chebyshev polynomial method was faster than that for the Padé and LS methods. In the ATM studies, the computation time taken by the Chebyshev method was three times smaller than in the Padé method and 30 times smaller than in the LS method, for any approximation order.

Hiliuta *et al.* [15] applied a combination of 'pchip' and 'fuzzy clustering techniques' for the interpolation of unsteady forces calculated for a range of non-evenly spaced reduced frequencies. However, if the range of reduced frequencies was evenly spaced, the results were obtained using the LS method. With this new method (efficient mainly for a range of non-evenly reduced frequencies), the approximations of these unsteady generalized forces remained in the frequency domain, and in order to obtain their approximations in the Laplace domain, it was necessary to apply a classical method such as LS or MS.

Botez *et al.* [16] presented a new mixed method that combines the LS and MS methods. This new method gave very good results with respect to the LS method and combined the strengths of the two classical methods, LS and MS. Flutter analysis results were presented for a business CL-604 aircraft.

In this article, a new method called the corrected minimum state (CMS) is presented. This method is based on the MS method. The error is estimated and approximated with the same analytical form as the MS standard form. Both forms are further combined to give the final CMS approximation for aerodynamic forces. It was found that the new CMS method gives better results in terms of accuracy than the MS method.

2 METHOD PRESENTATION

In order to present the algorithm for the new method, the approximation of unsteady aerodynamic forces used in the LS method has been introduced

$$\hat{Q}_{\mathrm{LS}}(s) = \mathbf{A}_0^{\mathrm{LS}} + \mathbf{A}_1^{\mathrm{LS}} s + \mathbf{A}_2^{\mathrm{LS}} s^2 + \sum_{i=1}^{n_{\mathrm{Lags}}} \frac{\mathbf{A}_{i+2}^{\mathrm{LS}}}{s - b_i} s \tag{1}$$

where b_i are the lags terms and n_{Lags} represents the total number of lag terms.

The MS algorithm approximates the aerodynamic forces $Q_{ij}(k)$ calculated in the frequency domain into the Laplace domain using the following equation

$$\hat{Q}_{\mathrm{MS}}(s) = \mathbf{A}_0^{\mathrm{MS}} + \mathbf{A}_1^{\mathrm{MS}} s + \mathbf{A}_2^{\mathrm{MS}} s^2 + \mathbf{D}^{\mathrm{MS}} \left[sI - \mathbf{R}^{\mathrm{MS}}\right]^{-1} \mathbf{E}^{\mathrm{MS}} s \tag{2}$$

where $\mathbf{A}_{0,1,2}^{\mathrm{MS}}$ is an assembly of estimated matrices of $(n \times m)$ dimensions, $\mathbf{R}$ is the diagonal square matrix of dimension $(n_{\mathrm{Lags}} \times n_{\mathrm{Lags}})$, $\mathbf{D}^{\mathrm{MS}}$ and $\mathbf{E}^{\mathrm{MS}}$ matrices have $(n \times n_{\mathrm{Lags}})$ and $(n_{\mathrm{Lags}} \times m)$ dimensions, and n_{Lags} represents the total number of lag terms.

The disadvantage of the LS method is that a larger number of states are used than in the MS method in the final system form [5].

The MS method with a large number of lags is more accurate than the MS method with a small number of lags. However, the MS method with a larger number of lags takes longer execution time than the MS method with a small number of lags. The results obtained with the CMS method will be close to the results obtained with the MS method with large number of lags; therefore, accuracy and execution time by use of the CMS method will be improved.

To estimate the values of $\mathbf{A}_{0,1,2}^{\mathrm{MS}}$, $\mathbf{D}$, $\mathbf{R}$, and $\mathbf{E}$ matrices, the classical MS algorithm minimizes the quadratic error criteria defined as follows

$$J = \sum_i \sum_j \sum_l W_{ijl}^2 \left| Q_{ij}(jk_l) - \hat{Q}_{ij,\mathrm{MS}}(jk_l) \right|^2 \tag{3}$$

where i and j are the indices of rows and columns and l is the index of reduced frequencies k's.

By solving equation (2) and by optimization of the J function, the MS method will give the unsteady forces approximation in the Laplace domain $\hat{Q}_{\mathrm{MS}}(s)$ for the unsteady forces in the reduced frequency domain $Q(k)$. The difference between the unsteady aerodynamic forces calculated in the frequency domain by the Doublet Lattice Method in the subsonic regime and by the Constant Pressure Method in the supersonic regime by use of Nastran software written as $Q(k)$ and the unsteady aerodynamic forces in the Laplace domain s gives the matrix error **Err** (s) for one Mach number

$$\mathbf{Err}(s) = Q(k) - Q_{\mathrm{MS}}(s) \tag{4}$$

$\mathbf{Err}(s)$ represents a function of s, which quantifies the difference between the estimated aerodynamic forces and the forces initially calculated in the frequency domain. Then, this function $\mathbf{Err}(s)$ is written under the same analytical form as the approximated aerodynamic forces by the MS method as shown in equation (2), and one obtains

$$\mathbf{Err}(s) = \mathbf{A}_0^{\mathrm{Err}} + \mathbf{A}_1^{\mathrm{Err}} s + \mathbf{A}_2^{\mathrm{Err}} s^2 + \mathbf{D}^{\mathrm{Err}} \left[sI - \mathbf{R}^{\mathrm{Err}}\right]^{-1} \mathbf{E}^{\mathrm{Err}} s \tag{5}$$

One assumption relates three matrices given in equation (5) to three matrices given in equation (2) as follows: $\mathbf{D}^{\mathrm{Err}} = \mathbf{D}^{\mathrm{MS}}$, $\mathbf{R}^{\mathrm{Err}} = \mathbf{R}^{\mathrm{MS}}$ (**D** and **R** matrices are the same as the ones given by the MS method), and $\mathbf{E}^{\mathrm{Err}} = \mathbf{E}^{\mathrm{MS}} \times \mathbf{E}^*$. Thus, equation (5) becomes

$$\mathbf{Err}(s) = \mathbf{A}_0^{\mathrm{Err}} + \mathbf{A}_1^{\mathrm{Err}} s + \mathbf{A}_2^{\mathrm{Err}} s^2 + \mathbf{D}^{\mathrm{MS}} \left[sI - \mathbf{R}^{\mathrm{MS}}\right]^{-1} \mathbf{E}^{\mathrm{MS}} \mathbf{E}^* s \tag{6}$$

where $\mathbf{E}^*$ is assumed to have $(n \times m)$ dimensions, in order to simplify the notations in the above equations.

In order to simplify the method, it has been assumed that the error, calculated with the MS method, affects only the elements of $\mathbf{A}_{0,1,2}^{\mathrm{Err}}$ and $\mathbf{E}^*$ matrices, whereas the elements of $\mathbf{D}^{\mathrm{MS}}$ and $\mathbf{R}^{\mathrm{MS}}$ matrices are not subject to any error. The product $\mathbf{D}^{\mathrm{MS}}\left[sI - \mathbf{R}^{\mathrm{MS}}\right]^{-1}\mathbf{E}^{\mathrm{MS}}\mathbf{E}^*$ is subject to errors propagated by the matrix 'error' $\mathbf{E}^*$. However, the reader may assume that the errors are made simultaneously on the **D** and **E** matrices. The problem reduces to find the values of $\mathbf{A}_{0,1,2}^{\mathrm{Err}}$ and $\mathbf{E}^*$ matrices. These matrices are computed by minimizing the new criteria defined as follows

$$J = \sum_{i=1}^{n_k} \left| \mathbf{Err}_i - \mathbf{A}_0^{\mathrm{Err}} - \mathbf{A}_1^{\mathrm{Err}} s - \mathbf{A}_2^{\mathrm{Err}} s^2 - \mathbf{D}^{\mathrm{MS}} \left[sI - \mathbf{R}^{\mathrm{MS}}\right]^{-1} \mathbf{E}^{\mathrm{MS}} \mathbf{E}^* s \right|^2 \tag{7}$$

where n_k represents the total number of reduced frequencies.

The final system can be written as $\mathbf{B} = \mathbf{A}\,\mathbf{x}$ by deriving the criteria J with respect to $\mathbf{A}_{0,1,2}^{\mathrm{Err}}$ and $\mathbf{E}^*$ matrices, written as $\partial J/\partial \mathbf{A}_{0,1,2}^{\mathrm{Err}} = 0$ and $\partial J/\partial \mathbf{E}^* = 0$. Matrices **B**, **A**, and **x** are

$$\mathbf{B} = \left[\sum_1^{n_{\mathrm{Lags}}} \mathbf{Err} \quad \sum_1^{n_{\mathrm{Lags}}} s \ \ \mathbf{Err} \sum_1^{n_{\mathrm{Lags}}} s^2 \ \ \mathbf{Err} \quad \mathbf{D} \sum_1^{n_{\mathrm{Lags}}} [sI - \mathbf{R}]^{-1} \mathbf{E} s \ \ \mathbf{Err} \right]^{\mathrm{T}}$$

$$\mathbf{A} = \begin{bmatrix} \sum_1^{n_{\mathrm{Lags}}} 1 & \sum_1^{n_{\mathrm{Lags}}} s & \sum_1^{n_{\mathrm{Lags}}} s^2 & \mathbf{D} \sum_1^{n_{\mathrm{Lags}}} [sI - \mathbf{R}]^{-1} s\mathbf{E} \\ \sum_1^{n_{\mathrm{Lags}}} s & \sum_1^{n_{\mathrm{Lags}}} s^2 & \sum_1^{n_{\mathrm{Lags}}} s^3 & \mathbf{D} \sum_1^{n_{\mathrm{Lags}}} [sI - \mathbf{R}]^{-1} s^2\mathbf{E} \\ \sum_1^{n_{\mathrm{Lags}}} s^2 & \sum_1^{n_{\mathrm{Lags}}} s^3 & \sum_1^{n_{\mathrm{Lags}}} s^4 & \mathbf{D} \sum_1^{n_{\mathrm{Lags}}} [sI - \mathbf{R}]^{-1} s^3\mathbf{E} \\ \mathbf{D} \sum_1^{n_{\mathrm{Lags}}} [sI - \mathbf{R}]^{-1} s\mathbf{E} & \mathbf{D} \sum_1^{n_{\mathrm{Lags}}} [sI - \mathbf{R}]^{-1} s^2\mathbf{E} & \mathbf{D} \sum_1^{n_{\mathrm{Lags}}} [sI - \mathbf{R}]^{-1} s^3\mathbf{E} & \mathbf{D} \sum_1^{n_{\mathrm{Lags}}} [sI - \mathbf{R}]^{-1} sE \end{bmatrix}$$

and

$$\mathbf{x} = \left[\mathbf{A}_0^{\mathrm{Err}} \quad \mathbf{A}_1^{\mathrm{Err}} \quad \mathbf{A}_2^{\mathrm{Err}} \quad \mathbf{E}^* \right]^{\mathrm{T}} \tag{8}$$

Finally, the total estimated aerodynamic forces can be expressed by equation (9)

$$Q(s) = \mathbf{A}_0^{\mathrm{tot}} + \mathbf{A}_1^{\mathrm{tot}} s + \mathbf{A}_2^{\mathrm{tot}} s^2 + \mathbf{D}^{\mathrm{tot}} \left[sI - \mathbf{R}^{\mathrm{tot}}\right]^{-1} \mathbf{E}^{\mathrm{tot}} s \tag{9}$$

Table 1 Relative numerical errors of the first flutter speeds and frequencies calculated by the MS and CMS methods for various lag terms versus the standard *pk* method, for Mach number = 1.1 and F/A-18 symmetric modes

	MS method		CMS method	
Number of lag terms	EAS_1 (%)	First frequency (%)	EAS_1 (%)	First frequency (%)
2	1.97	1.52	1.52	1.36
3	0.44	1.85	0.65	1.69
4	0.28	1.52	0.10	0.72
5	0.06	1.28	0.20	1.36
6	0.26	1.04	0.25	0.88
7	0.03	0.80	0.06	0.64
8	0.22	0.64	0.27	0.56
9	0.29	1.44	0.19	1.36
10	0.16	0.80	0.17	0.72

where

$$\mathbf{A}_{0,1,2}^{\text{tot}} = \mathbf{A}_{0,1,2}^{\text{MS}} + \mathbf{A}_{0,1,2}^{\text{Err}}$$
$$\mathbf{D}^{\text{tot}} = \mathbf{D}^{\text{MS}}$$
$$\mathbf{R}^{\text{tot}} = \mathbf{R}^{\text{MS}}$$
$$\mathbf{E}^{\text{tot}} = \mathbf{E}^{\text{MS}}(I + \mathbf{E}^{*})$$

3 RESULTS AND DISCUSSIONS

Using the approximation of the aerodynamic forces with the MS and CMS methods in the *pk* flutter method, several flutter point results expressed in terms of flutter speeds and frequencies were obtained. Both methods were applied on an F/A-18 aircraft, on its symmetrical and anti-symmetrical modes, for two Mach numbers $M = 1.1$ and $M = 1.3$.

Table 2 Relative numerical errors of the second flutter speeds and frequencies calculated by the MS and CMS methods for various lag terms versus the standard *pk* method, for Mach number = 1.1 and F/A-18 symmetric modes

	MS method		CMS method	
Number of lag terms	EAS_2 (%)	Second frequency (%)	EAS_2 (%)	Second frequency (%)
2	10.25	1.39	3.42	0.51
3	11.86	1.49	5.09	0.77
4	10.88	1.28	4.80	0.62
5	8.93	0.91	5.17	0.66
6	5.26	0.55	3.17	0.29
7	3.63	0.33	2.15	0.18
8	2.82	0.26	1.81	0.11
9	4.64	0.55	2.85	0.29
10	2.44	0.18	1.45	0.11

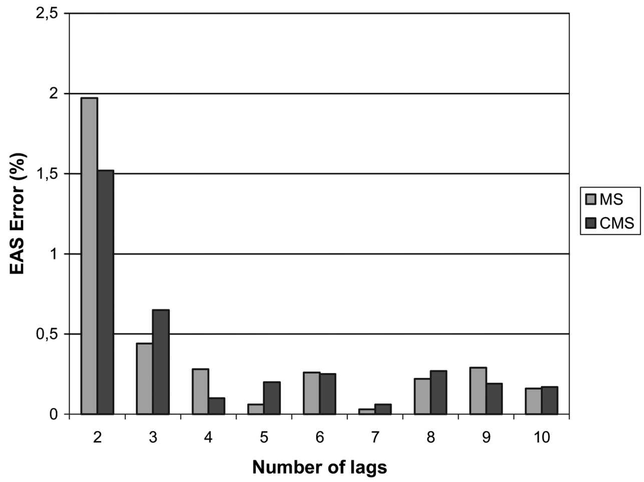

Fig. 1 Results of Table 1 visualized under bars form

As comparison criteria, equations (10a) and (10b) are used, where the error reduction rate and abs, which is the absolute value function, are defined as follows

$$100 * \frac{\text{abs}(V_{\text{MS}} - V_{\text{pk}})}{V_{\text{pk}}} \quad 100 * \frac{\text{abs}(f_{\text{MS}} - f_{\text{pk}})}{V_{\text{pk}}} \tag{10a}$$

$$100 * \frac{\text{abs}(V_{\text{CMS}} - V_{\text{pk}})}{V_{\text{pk}}} \quad 100 * \frac{\text{abs}(f_{\text{CMS}} - f_{\text{pk}})}{V_{\text{pk}}} \tag{10b}$$

Tables 1 and 2 show the relative errors of the first and second flutter speeds and frequencies calculated by the MS and CMS methods for a set of two to ten lag terms with respect to the first and second flutter speeds and frequencies calculated by the *pk* standard method for the symmetric modes of an F/A-18 aircraft at Mach number = 1.1.

Figures 1 and 2 show visually, in the form of bars, the numerical results presented in Tables 1 and 2.

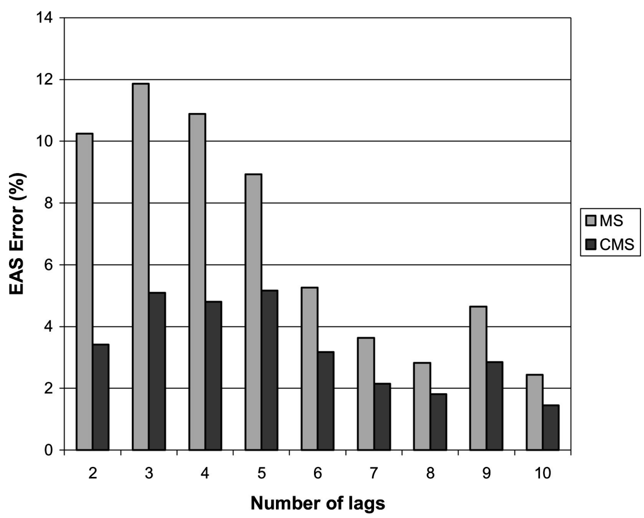

Fig. 2 Results of Table 2 visualized under bars form

Table 3 Relative numerical errors of the first flutter speeds and frequencies calculated by the MS and CMS methods for various lag terms versus the standard *pk* method, for Mach number = 1.1 and F/A-18 anti-symmetric modes

	MS method		CMS method	
Number of lag terms	EAS_1 (%)	First frequency (%)	EAS_1 (%)	First frequency (%)
2	0.66	0.29	0.40	0.14
3	0.77	0.36	0.65	0.22
4	0.49	0.07	0.40	0
5	0.40	0.29	0.32	0.29
6	0.06	0.43	0.01	0.29
7	0.21	0.43	0.17	0.36
8	0.03	0.36	0.003	0.29
9	0.21	0.29	0.21	0.29
10	0.06	0.07	0.01	0.22

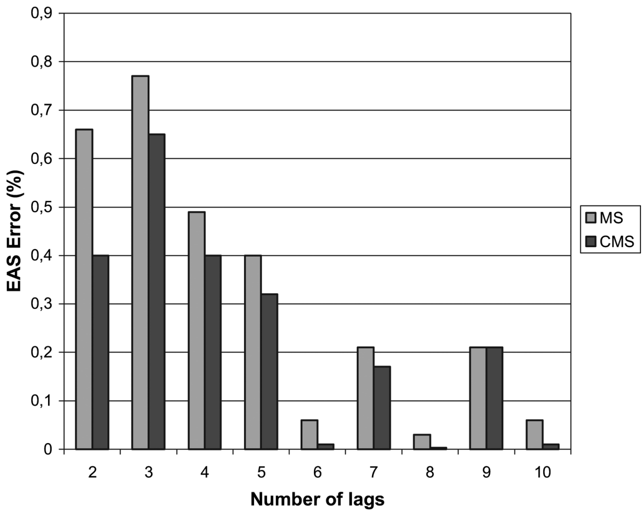

Fig. 3 Results of Table 3 visualized under bars form

Tables 3 and 4 show the relative errors of the first and second flutter speeds and frequencies calculated by the MS and CMS methods for a set of two to ten lag terms with respect to the first and second flutter speeds and frequencies calculated by the *pk* standard method for the anti-symmetric modes of an F/A-18 aircraft at Mach number = 1.1. Figures 3 and 4 show visually, in the form of three-dimensional bars, the numerical results presented in Tables 3 and 4.

Tables 5 and 6 show the relative errors of the first and second flutter speeds and frequencies calculated by the MS and CMS methods for a set of two to ten lag terms with respect to the first and second flutter speeds and frequencies calculated by the *pk* standard method for the symmetric modes of an F/A-18 aircraft at Mach number = 1.3. Figures 5 and 6 show visually, in the form of bars, the numerical results presented in Tables 5 and 6.

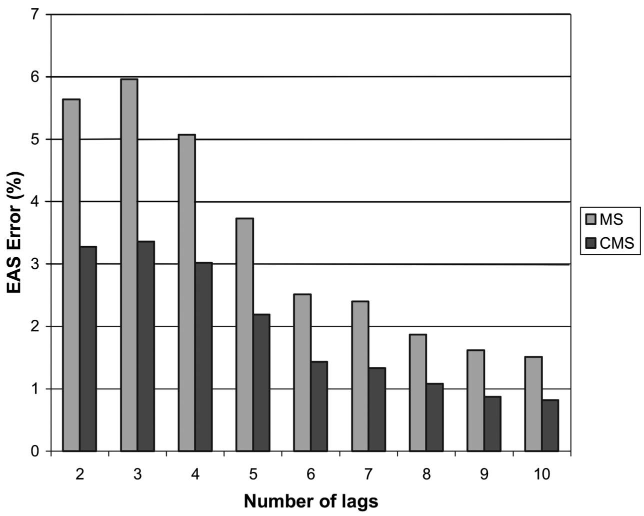

Fig. 4 Results of Table 4 visualized under bars form

Table 4 Relative numerical errors of the second flutter speeds and frequencies calculated by the MS and CMS methods for various lag terms versus the standard *pk* method, for Mach number = 1.1 and F/A-18 anti-symmetric modes

	MS method		CMS method	
Number of lag terms	EAS_2 (%)	Second frequency (%)	EAS_2 (%)	Second frequency (%)
2	5.64	0.44	3.28	0.40
3	5.96	0.58	3.36	0.40
4	5.07	0.51	3.02	0.36
5	3.73	0.33	2.19	0.22
6	2.51	0.26	1.43	0.11
7	2.40	0.26	1.33	0.07
8	1.87	0.15	1.08	0.07
9	1.62	0.11	0.87	0.04
10	1.51	0.11	0.82	0.04

Table 5 Relative numerical errors of the first flutter speeds and frequencies calculated by the MS and CMS methods for various lag terms versus the standard *pk* method, for Mach number = 1.3 and F/A-18 symmetric modes

	MS method		CMS method	
Number of lag terms	EAS_1 (%)	First frequency (%)	EAS_1 (%)	First frequency (%)
2	1.25	1.31	0.37	0.07
3	0.56	0	0.17	0
4	0.27	0.04	0.08	0
5	0.24	0.04	0.03	0.04
6	0.16	0	0.04	0
7	0.16	0.04	0.04	0.04
8	0.09	0.04	0.10	0

Table 6 Relative numerical errors of the second flutter speeds and frequencies calculated by the MS and CMS methods for various lag terms versus the standard *pk* method, for Mach number = 1.3 and F/A-18 symmetric modes

	MS method		CMS method	
Number of lag terms	EAS_2 (%)	Second frequency (%)	EAS_2 (%)	Second frequency (%)
2	6.4	2.14	5.33	0.29
3	3.04	0.29	2.22	0.29
4	1.17	0.14	0.06	0.07
5	1.67	0.21	1.59	0.14
6	0.72	0.14	0.72	0.07
7	0.85	0	0.75	0.07
8	1.28	0.14	1.28	0.07

Table 7 shows the relative errors of the first flutter speed and frequency calculated by the MS and CMS methods for a set of two to eight lag terms with respect to the first flutter speed and frequency calculated by the *pk* standard method for the anti-symmetric modes of an F/A-18 aircraft at Mach number = 1.3. Figure 7 shows visually, in the form of bars, the numerical results presented in Table 7.

4 CONCLUSIONS

From Tables 1, 3, 5, and 7 for F/A-18 aircraft symmetric modes, the smallest first equivalent airspeed EAS percentage calculated with the MS method appears for seven lag terms (=0.03). The first closest EAS percentage (calculated by the CMS method) to this value was found for the same number of seven lag terms (=0.06). The same criterion of reasoning is applied for all other speeds and frequencies.

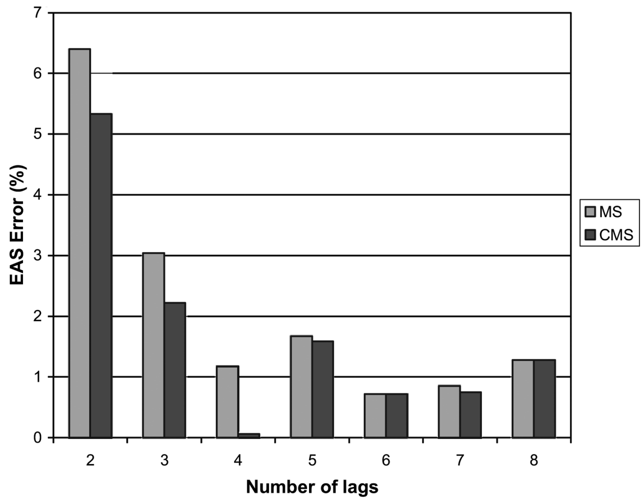

Fig. 6 Results of Table 6 visualized under bars form

Table 7 Relative numerical errors of the first flutter speeds and frequencies calculated by the MS and CMS methods for various lag terms versus the standard *pk* method, for Mach number = 1.3 and F/A-18 anti-symmetric modes

	MS method		CMS method	
Number of lag terms	EAS_1 (%)	First frequency (%)	EAS_1 (%)	First frequency (%)
2	1.53	0.32	0.57	0.18
3	1.26	0.25	0.83	0.22
4	0.65	0.11	0.63	0.11
5	0.31	0.04	0.23	0.04
6	0.14	0	0.05	0
7	0.22	0.07	0.17	0.04
8	0.21	0.07	0.20	0.07

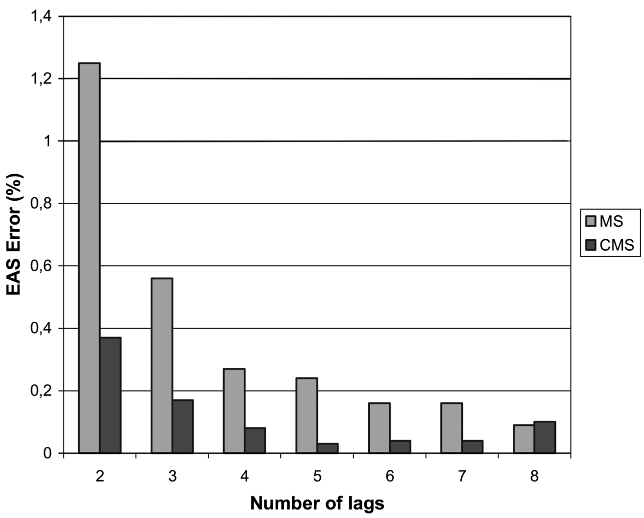

Fig. 5 Results of Table 5 visualized under bars form

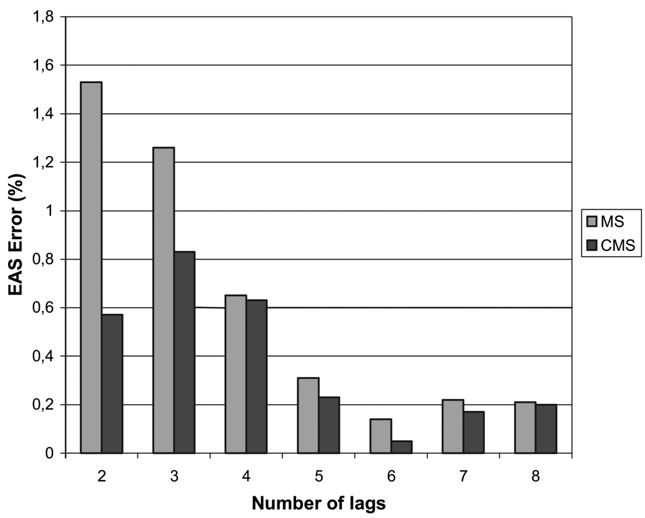

Fig. 7 Results of Table 7 visualized under bars form

Table 8 Number of optimal lag terms corresponding to the closest first equivalent airspeeds and frequencies calculated by the MS and CMS methods for four cases: $M = 1.1$ and $M = 1.3$ for symmetric and anti-symmetric modes of an F/A-18 aircraft

	MS method		CMS method	
Mach number and modes type	EAS_1 (%)	First frequency (%)	EAS_1 (%)	First frequency (%)
$M = 1.1$ symmetric	7	8	7	7
$M = 1.1$ anti-symmetric	8	4	6	2
$M = 1.3$ symmetric	8	3	4	3
$M = 1.3$ anti-symmetric	6	7	6	4

As shown in Table 8, which summarizes the results shown in Tables 1, 3, 5, and 7, the number of optimal lag terms was found to be always smaller in the case of the new CMS method application in comparison with the number of optimal lag terms used in the MS method. Table 9 summarizes the results given in Tables 2, 4, and 6 for the second flutter speeds and frequencies.

To obtain the closest flutter speeds and frequencies to the flutter speeds and frequencies calculated by the *pk* standard method, the new method requires a fewer number of lag terms than the classical LS method, as seen in Tables 8 and 9.

For this reason, the first and main advantage of this new method with respect to the classical MS method is its execution speed, which is faster as fewer lag terms are necessary to obtain the flutter speed closest to the *pk* flutter speed.

The form of aerodynamic forces approximated with the new method is similar to the form of these forces approximated with the classical MS method. For this reason, the second advantage of this new method consists in its integration into the aeroelastic equations of motion, which is similar to the integration of the MS classical method into these equations. An aspect of its integration consists in the evaluation of the approximation error and its addition to the final approximation error form.

Table 9 Number of optimal lag terms corresponding to the closest second equivalent airspeeds and frequencies calculated by the MS and CMS methods for four cases: $M = 1.1$ and $M = 1.3$ for symmetric and anti-symmetric modes of an F/A-18 aircraft

	MS method		CMS method	
Mach number and modes type	EAS_2 (%)	Second frequency (%)	EAS_2 (%)	Second frequency (%)
$M = 1.1$ symmetric	10	10	7	7
$M = 1.1$ anti-symmetric	10	9	6	6
$M = 1.3$ symmetric	6	7	6	4
$M = 1.3$ anti-symmetric	–	–	–	–

For an F/A-18 aircraft, the new method developed in this article based on the classical MS method was found to be very efficient for aeroservoelastic interaction studies.

This fact can be explained by the fact that the MS method becomes more accurate when a higher number of lag terms are added, which imply high amounts of calculations affecting the speed of computations, whereas the new method reduces the execution time, which is important for the aeroservoelasticity studies on an aircraft.

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APPENDIX

Notation

$\mathbf{A}^{MS}_{0,1,2}$	estimated matrix by the MS method of $(r \times c)$ dimensions
$\mathbf{A}^{Err}_{0,1,2}$	error matrix of $(r \times c)$ dimensions
$\mathbf{D}^{MS}$	estimated matrix by the MS method of $(r \times n_{Lags})$
$\mathbf{D}^{Err}$	error matrix of $(r \times n_{Lags})$
$\mathbf{E}^{MS}$	estimated matrix by the MS method of $(n_{Lags} \times c)$
$\mathbf{E}^{Err}$	error matrix of $(n_{Lags} \times c)$
$\mathbf{E}^{*}$	error matrix term in $\mathbf{E}^{Err}$ matrix of dimension $(c \times c)$
$\mathrm{Err}(s)$	estimated error matrix in the Laplace domain
J	quadratic optimization criteria
k_i	reduced frequencies
n_{Lags}	total number of lag terms
$Q(k)$	aerodynamic forces in the frequency domain
$Q_{ij}(k)$	aerodynamic force element in the frequency domain
$Q(s)$	aerodynamic forces in the Laplace domain
$\hat{Q}_{MS}(s)$	aerodynamic forces estimated by the MS method in the Laplace domain
$\mathbf{R}^{MS}$	diagonal square matrix of dimension $(n_{Lags} \times n_{Lags})$
$\mathbf{R}^{Err}$	error diagonal square matrix of dimension $(n_{Lags} \times n_{Lags})$