Improved Method for Creating Time-Domain Unsteady Aerodynamic Models

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Abstract: Multidisciplinary aeroservoelastic interactions are studied by the combination of knowledge acquired in two main disciplines: aeroelasticity and servocontrols. In aeroelasticity, the doublet lattice method is used to calculate the unsteady aerodynamic forces for a range of reduced frequencies and Mach numbers on a business aircraft in the subsonic flight regime by use of NASTRAN software. For aeroservoelasticity studies, there is the need to conceive methods for these unsteady aerodynamic forces conversions from frequency into Laplace domain. A new method different from classical methods is presented, in which Chebyshev polynomials theories and their orthogonality properties are applied. In this paper, a comparison between flutter results expressed in terms of flutter speeds and frequencies obtained with our method with flutter results obtained with classical Padé and least squares methods is presented for a business aircraft at one Mach number and a range of reduced frequencies. It has been found that results obtained with our method are better in terms of average error than results obtained with the two classical methods here presented.

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Introduction

Unsteady aerodynamic forces for a range of reduced frequencies and Mach numbers are usually calculated in the subsonic regime by use of the doublet lattice method (DLM) implemented in aeroelastic analyses software such as NASTRAN (Rodden et al. 1979), ADAM (Noll et al. 1986), STARS (Gupta 1997), and FAMUSS (Pitt 1992). For aeroservoelasticity studies, these unsteady aerodynamic forces calculated in the frequency domain should be converted into Laplace domain. In this paper, a new method is applied for this type of conversion.

The following classical methods: least squares (LS), matrix Padé, and minimum state (MS), are the best known methods for the conversion of the unsteady aerodynamic forces from frequency into Laplace domain (Karpel 1982). All the abovementioned aeroservoelastic software (except FAMUSS) mainly use these methods or their extended versions (Dunn 1980) such as extended LS, extended MP, and extended MS. Several unsteady aerodynamic forces approximations by use of the MS methods for several fixed Mach numbers were conceived (Poirion 1996). Four different order reduction methods were applied (Cotoi and Botez 2002) for the last term of the LS approximation. The value of the error obtained with the best chosen method among these four methods was found to be 12–40 times smaller than the value of the error obtained with the MS classical method. The disadvantage of this method was its computing time, which is higher than the computing time taken by the MS method for these approximations. Therefore, the computing time of flutter frequencies and speeds was higher too. Smith showed that the P - transform and the FAMUSS methods generate state space system approximations directly (Smith et al. 2004). For the calculations of unsteady aerodynamic forces approximations for any range of reduced frequencies, a new method was used by a combination of fuzzy clustering with shape-preserving techniques (Hiliuta et al. 2005). A new method called mixed state, which combined the analytical expressions given by the LS and the MS methods, was presented (Biskri et al. 2005).

A new method based on the Chebyshev polynomials and their orthogonality properties is described in this paper and this new method was applied to a business aircraft model with 44 symmetric modes and 50 antisymmetric modes, the same model as that treated by Biskri et al. (2005). Results obtained by use of this new method were found to be better than results obtained with the classical Padé or LS methods.

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Equations of Motion of an Aircraft

The classical equations of motion of an aircraft are expressed as follows:

$$\mathbf{M}\ddot{\eta} + \mathbf{C}\dot{\eta} + \mathbf{K}\eta + q\mathbf{Q}(k,\mathrm{M})\eta = P(t) \quad (1)$$

which is written as function of generalized coordinates η. Structural matrices are the generalized mass, damping, and stiffness matrices. In Eq. (1), $q=\rho V^2/2$=dynamic pressure with ρ as the air density and V as the true airspeed; $P(t)$=external force due to gusts, turbulence, or pilot inputs on aircraft control surfaces; and $k=\omega b/V$=reduced frequency, where ω=oscillations frequency and b=wing semichord length.

For aeroservoelastic interactions studies, there is the need to obtain the aerodynamic forces in the Laplace domain. Therefore, Eq. (1) will be converted in the Laplace domain

$$[\mathbf{M}s^2 + \mathbf{C}s + \mathbf{K}]\eta(s) + q\mathbf{Q}(s)\eta(s) = 0 \quad (2)$$

A new method used to convert the aerodynamic forces $Q(k,M)$ from frequency into Laplace domain $\mathbf{Q}(s)$ is described in this paper. The method uses the Chebyshev polynomials and their orthogonality property (Weisstein 2004).

Aerodynamic Forces Calculations by Use of the Chebyshev Method

For aeroelasticity studies, the unsteady generalized aerodynamic forces $Q(i,j)$ are calculated by the DLM in NASTRAN, where $i,j=1,2,\ldots,50$ for the CL-604 antisymmetric 50 modes and $i,j=1,2,\ldots,44$ for the CL-604 symmetric 44 modes. These forces are calculated on a CL-604 for one Mach number and a set of reduced frequencies.

Predefined Chebyshev functions, such as *chebpade* and *chebyshev*, defined in MATLAB allow the construction of a polynomial interpolation for the unsteady generalized aerodynamic forces from frequency to Laplace domain.

The *chebyshev* function is used for the unsteady aerodynamic force matrix approximations under the following power series form:

$$\mathbf{Q}_{ij}(s) = \frac{1}{2}c_0^{(ij)} + \sum_{n=1}^{\infty} c_n^{(ij)} T_n^{(ij)}(s) \quad (3)$$

where coefficients of this approximation are

$$c_n^{(ij)} = \frac{2}{\pi}\int_{-1}^{1} \frac{\mathbf{Q}_{ij}(s)T_n^{(ij)}(s)}{\sqrt{(1-s^2)}}ds$$

for $n=0,1,\ldots$.

The *chebpade* function is used for the unsteady aerodynamic force matrix approximations under the following rational fractions form:

$$\mathbf{Q}_{ij}(s) = \frac{\sum_{n=0}^{P+2} a_n^{(ij)} T_n^{(ij)}(s)}{1 + \sum_{n=1}^{P} b_n^{(ij)} T_n^{(ij)}(s)} \quad (4)$$

where the numerator degree is greater than the denominator degree by a factor of 2. Thus, the order of the Chebyshev polynomials can be written under the form $[P+2,P]$ and this order will be varied in the Results section in order to see the differences appearing between results obtained for different orders of Chebyshev polynomials.

Flutter Analyses

In order to demonstrate the flutter analysis theory used in this paper, we define the following ratios: the air density ratio σ, which is the ratio between the air density at a certain altitude ρ and the air density at the sea level ρ_0:

$$\sigma = \frac{\rho}{\rho_0} \quad (5)$$

Eq. (1) is used for flutter analysis where the aerodynamic unsteady forces matrix $\mathbf{Q}$ is complex and therefore $\mathbf{Q}$ has a real part $\mathbf{Q}_R$ and an imaginary part $\mathbf{Q}_I$. The aerodynamic stiffness $\mathbf{Q}_R$ is in phase with the vibration displacement and therefore is associated with η. The *aerodynamic damping* $\mathbf{Q}_I$ is in phase with the vibration velocity, and therefore is associated with $\dot{\eta}$. Thus, Eq. (1) may be written as follows:

$$\mathbf{M}\ddot{\eta} + \left(\mathbf{C} + \frac{1}{\omega}q\mathbf{Q}_I\right)\dot{\eta} + (\mathbf{K} + q\mathbf{Q}_R)\eta = 0 \quad (6)$$

From the reduced frequency k definition, ω is written as a function of k

$$\omega = kV/b \quad (7)$$

We replace ω given by Eq. (7) and $q=\rho V^2/2$ already expressed in the second section in Eq. (6), so that next equation is obtained

$$\mathbf{M}\ddot{\eta} + \left(\mathbf{C} + \frac{1}{2k}\rho Vb\mathbf{Q}_I\right)\dot{\eta} + \left(\mathbf{K} + \frac{1}{2k}\rho Vb\mathbf{Q}_R\right)\eta = 0 \quad (8)$$

From the dynamic pressure definition, we write

$$\rho V^2 = \rho_0 V_E^2 \quad (9)$$

The equivalent airspeed V_E is written as function of air density ratio σ

$$V_E = \sqrt{\sigma}V \quad (10)$$

Terms on both sides of Eq. (9) are divided by V, and by use of Eq. (10) we obtain

$$\rho V = \rho_0\frac{V_E^2}{V} = \rho_0 V_E\frac{V_E}{V} = \rho_0 V_E\sqrt{\sigma} \quad (11)$$

Eq. (8) is further written as a function of the equivalent airspeed V_E by use of Eqs. (9)–(11)

$$\mathbf{M}\ddot{\eta} + \left(\mathbf{C} + \frac{1}{2k}\rho_0 b\sqrt{\sigma}V_E\mathbf{Q}_I\right)\dot{\eta} + \left(\mathrm{K} + \frac{1}{2}\rho_0 V_E^2\mathbf{Q}_R\right)\eta = 0 \quad (12)$$

Results

A comparison is presented in this paper between the results obtained by use of our Chebyshev approximation method with the results obtained by two classical approximation methods such as the LS and Padé methods. Regarding these two classical methods, the LS method is very well known in aeroservoelastic interactions studies and was already applied in aircraft industry, while the Padé method uses a parameter identification solution in order to determine a polynomial fractional form which identifies an or-

Table 1. Total Normalized Errors

Mode type	Approximation order	$J_{Q\ \mathrm{real}}$ Padé	$J_{Q\ \mathrm{imag}}$ Padé	$J_{Q\ \mathrm{real}}$ Chebyshev	$J_{Q\ \mathrm{imag}}$ Chebyshev
Symmetric modes	[6, 4]	8.35821	9.4265	0.1637	0.1544
	[7, 5]	123.9417	139.7621	0.0354	0.0132
	[8, 6]	50.9315	96.2094	0.0354	0.0132
	[9, 7]	1.8054	1.1595	0.0354	0.0132
	[10, 8]	58.9544	65.7236	0.0354	0.0132
Antisymmetric modes	[6, 4]	15.6480	15.2306	0.3040	0.3024
	[7, 5]	n.o.	n.o.	0.0397	0.0192
	[8, 6]	n.o.	n.o.	0.0397	0.0192
	[9, 7]	n.o.	n.o.	0.0397	0.0192
	[10, 8]	n.o.	n.o.	0.0397	0.0192

thogonal polynomial interpolation. This fractional form is the key aspect of the Padé method, due to the fact that it allows the order reduction system.

These methods were applied for unsteady aerodynamic forces approximations on a business aircraft modeled by finite-elements methods with NASTRAN code at the Mach number $\mathrm{M}=0.88$ and at eight reduced frequencies $k=0.001$, 0.1, 0.3, 0.5, 0.7, 0.9, 1.1, and 1.4 and for different polynomial approximation orders.

The polynomial approximation order in the Chebyshev method such as [6, 4] represents the maximum rank of the Chebyshev polynomials used to form the numerator and the denominator in Eq. (4). Thus, in Eq. (4), an approximation order [6, 4] gives $P=4$ where $P+2=$maximum rank of Chebyshev polynomials at the numerator and $P=$maximum rank of Chebyshev polynomials at the denominator. The approximation order for Padé polynomials is defined in the same manner as the approximation order for Chebyshev polynomials.

Different other values of the polynomial approximation order by the Padé method and the Chebyshev polynomial fractions method (polynomial order should be equivalent for both methods) were used for the total normalized approximation error calculations—which were found to be much smaller for the Chebyshev polynomials method with respect to the overall approximation error given by Padé polynomials method. The approximation error for each element of the $\mathbf{Q}$ matrix is normalized for the real and the imaginary part, at each reduced frequency by use of

$$J_{\mathbf{Q}\ \mathrm{real}}=\sum_{k=1}^{14}\left(\sum_{i=1}^{N_{\mathrm{modes}}}\left(\sum_{j=1}^{N_{\mathrm{modes}}}\frac{|\mathbf{Q}_{ij\ R\ \mathrm{new}}-\mathbf{Q}_{ij\ R\ \mathrm{old}}|}{\sqrt{|\mathbf{Q}_{ij}|^2}}\right)\right)\times 100\% \tag{13a}$$

$$J_{\mathbf{Q}\ \mathrm{imaginary}}=\sum_{k=1}^{14}\left(\sum_{i=1}^{N_{\mathrm{modes}}}\left(\sum_{j=1}^{N_{\mathrm{modes}}}\frac{|\mathbf{Q}_{ij\ I\ \mathrm{new}}-\mathbf{Q}_{ij\ I\ \mathrm{old}}|}{\sqrt{|\mathbf{Q}_{ij}|^2}}\right)\right)\times 100\% \tag{13b}$$

and

$$J_{\mathbf{Q}}=J_{\mathbf{Q}\ \mathrm{real}}+J_{\mathbf{Q}\ \mathrm{imaginary}}$$

where $\mathbf{Q}_{R\ \mathrm{old}}$ and $\mathbf{Q}_{I\ \mathrm{old}}$ are the real and the imaginary parts of the unsteady aerodynamic forces given by NASTRAN and $\mathbf{Q}_{R\ \mathrm{new}}$ and $\mathbf{Q}_{I\ \mathrm{new}}=$real and the imaginary parts of the unsteady aerodynamic forces approximated by Padé or Chebyshev theories. $N_{\mathrm{modes}}=$total number of modes (also the dimension of $\mathbf{Q}$); $k=$index of the reduced frequency; and $J=$total normalized error. In Table 1 we compare the numerical values of these errors obtained for five different approximation orders: [6, 4], [7, 5], [8, 6], [9, 7], and [10, 8]. In Table 1, the abbreviation n.o. means that no results were obtained due to the fact that the approximation requires more than 1 Gbyte of memory, which is more memory than MATLAB 6.5 can handle. As shown in Table 1, the Chebyshev method gives a smaller error than the Padé method. In most of the cases, the normalized error given by Chebyshev method can be up to 100 times smaller than the error given by the Padé approximation method. In other cases, use of the Padé method is not efficient because of the amount of memory requested, which is too high for MATLAB 6.5 use. The second type of results comparison is given in terms of numerical values for flutter equivalent airspeeds (EAS) and frequencies ω and their errors J_{EAS} and J_{ω}, which are expressed by the following:

$$J_{\mathrm{EAS}}=\frac{|\mathrm{EAS}_{pk\_\mathrm{std}}-\mathrm{EAS}_{pk\_\mathrm{approx}}|}{\mathrm{EAS}_{pk\_\mathrm{std}}}\times 100\% \tag{14a}$$

Table 2. Flutter Errors J (%) for Business Aircraft with 50 Antisymmetric Modes

	F#2 Mode 9		F#3 Mode 10		F#5 Mode 17		F#7 Mode 66		
Method	J_{VE}	J_{Freq}	J_{VE}	J_{Freq}	J_{VE}	J_{Freq}	J_{VE}	J_{Freq}	Average error J
pk-LS 8 lags	0.55	0.16	1.00	1.21	4.87	0.37	1.03	0.34	1.19
pk-LS 10 lags	0.67	0	1.67	1.32	6.50	0.46	2.34	0.18	1.64
pk-Chebyshev	1.50	0.32	0.09	0	0.29	0.09	0.19	0	0.31

Table 3. Flutter Errors J (%) for Business Aircraft with 44 Symmetric Modes

Method	F#1 Mode 7		F#4 Mode 15		F#6 Mode 19		F#8 Mode 41		Average error
	J_{Veas}	J_{Freq}	J_{Veas}	J_{Freq}	J_{Veas}	J_{Freq}	J_{Veas}	J_{Freq}	J
pk-LS 8 lags	4.70	0.84	0.47	0.42	0.48	0.07	0.38	0.07	0.93
pk-LS 10 lags	4.23	0.42	5.92	1.53	0.06	0.15	0.24	0	1.57
pk-Chebyshev	0.20	0.14	0.21	0	0.05	0	0.08	0.04	0.09

$$J_{\omega} = \frac{|\omega_{pk\_\text{std}} - \omega_{pk\_\text{approx}}|}{\omega_{pk\_\text{std}}} \times 100\% \tag{14b}$$

where, in equation (14*a*), $\text{EAS}_{pk\_\text{std}}$ and ω_{pk_std}=equivalent airspeeds and frequencies calculated by the *pk standard method* presented earlier, whereas $\text{EAS}_{pk\_\text{approx}}$ and $\omega_{pk\_\text{approx}}$=equivalent airspeeds and frequencies calculated with the *pk_approximation method*, which is one of the two methods: *pk*-LS method (with eight and ten lag terms) and *pk*-Chebyshev method. The total average error is expressed by

$$J = \frac{\sum_{i=1}^{N_{\text{flutter}}} [J_{\text{EAS}}(i) + J_{\omega}(i)]}{2N_{\text{flutter}}} \tag{15}$$

where N_{flutter}=number of flutter points detected. For the business aircraft here presented, a number of four flutter points were detected, and the corresponding modes to these flutter points are given in Tables 2 and 3. In Tables 2 and 3, the flutter errors J [see Eqs. (14) and (15)] are presented in percentage values for the flutter equivalent airspeeds and frequencies for business aircraft with a number of 50 antisymmetric modes (Table 2) and for 44 symmetric modes (Table 3). From Tables 2 and 3, the average error was found to be very much smaller in the case of *pk*-Chebyshev method application than in case of *pk*-LS method application, which demonstrates the superiority of the Cheyshev method with respect to the *pk*-LS method.

Conclusions

Generally, an approximation method is considered to be better than another method if its approximation error is smaller and if its computation time is faster; and, ultimately, if less computer resources are used, which could be in some cases a crucial criterion (when a significantly larger quantity of data to be approximated is used). The case of the CL-604 from Bombardier proves to be in fact one of these cases, due to software limitations: The Padé approximation failed when we tried to use it for the 50 elastic antisymmetric mode case for model orders higher than [6, 4] because this approximation would require more than 1 Gbyte of memory, which is more than MATLAB 6.5 can handle.

Regarding the computation time, the Chebyshev method proved to be four times faster than Padé, depending on the number of modes and the model order used. The Chebyshev method, when compared with Padé, provided much smaller approximation errors, as shown in Table 1. A remarkable aspect regarding the Chebyshev method is that, using it on the CL-604's data, the total normalized approximation error obtained with this method rapidly converged (with the increase of the model order) to a constant very small value for both symmetric and antisymmetric mode cases, whereas the same approximation error provided by the Padé method was higher and presented large fluctuations. Due to this type of error, the Padé approximation was not able to provide all flutter points for the CL-604 when used in conjunction with the *pk* method, whereas the Chebyshev method provided these values with high accuracy. This is the reason why, to compare the flutter results of the Chebyshev method, we made appeal at the *pk*-LS method, a much slower, but accurate approximation method, based on Padé decomposition. However, even when compared to LS, the Chebyshev method, which proved to be up to 30 times faster than LS, provided smaller flutter average errors, no matter the number of lag terms that we have implemented when using the LS. Previous results obtained with the Chebyshev approximation method on the Aircraft Test Model data from STARS and the F/A-18 aircraft data kindly provided to us by NASA Dryden Flight Research Center and the above-presented results for the CL-604 aircraft data from Bombardier make us conclude that this approximation method is not problem dependent and that it is a fast, reliable, and very accurate method.

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Notation

The following symbols are used in this technical note:

$\mathbf{C}$ = modal damping matrix;
c = wing chord length;
$\mathbf{K}$ = modal elastic stiffness matrix;
k = reduced frequency;
$\mathbf{M}$ = modal inertia or mass matrix;
M = Mach number;
$\mathbf{Q}$ = modal generalized aerodynamic force matrix;
$\mathbf{Q}_I$ = imaginary part of modal generalized aerodynamic force matrix;
$\mathbf{Q}_R$ = real part of modal generalized aerodynamic force matrix;
q = dynamic pressure;
T = Chebyshev polynomial;
V = true airspeed;
V_E = equivalent airspeed;
V_0 = reference true airspeed;
η = generalized coordinates;
ν = airspeed ratio;
ρ = true air density;
ρ_0 = reference air density; and
ω = oscillations frequency.

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