

Engineering Notes

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Unsteady Aerodynamic Forces Mixed Method for Aeroservoelasticity Studies on an F/A-18 Aircraft

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I. Introduction

TWO classical methods are often used in the aeroservoelasticity studies for the conversion of unsteady aerodynamic forces from the frequency into Laplace domain. These two methods are the least squares (LS) and the minimum state (MS) [1–3]. These methods were also improved and modified.

Extended versions of the LS and MS methods were renamed extended LS methods (ELS) [4] and extended modified matrix Padé method (EMMP) [5]. In these extended versions, different restrictions were imposed to the aerodynamic force approximations to pass through certain points. These approximations should be exact in zero and in two other points which may represent the estimated flutter frequency and the estimated gust frequency.

The MS method was modified [6] to take into consideration two weighting types of aerodynamic generalized data. In the first weighting type, the aerodynamic generalized forces data were normalized to the maximum unit value of each aerodynamic coefficient. In the second weighting type, the effect of the incremental error of the aerodynamic coefficients on the aeroelastic characteristics of the system was considered. Qualities of the MS method [7] such as its generality, accuracy, and flexibility were found in all its applications on subsonic and supersonic aircraft.

Results obtained following the application of the MS method [8] in the equations of motion of an active flexible wing wind tunnel model showed that very good mathematical models may be obtained with fewer augmenting aerodynamic equations than traditional approaches by a factor of 10. This reduction facilitates the lower order control systems integration in aeroservoelastic systems.

Different modal based aeroservoelastic modeling techniques [9] were combined to conceive an integrated design optimization algorithm. To study the transient response of both open- and closed-

loop nonlinear aeroelastic systems, a generalized direct simulation method using a discrete time-domain state-space approach was developed.

Other aerodynamic force approximations were realized with several MS approximations by Poirion [10], for several Mach numbers and by use of spline interpolation methods.

The most known aeroservoelastic codes such as ADAM [11], ISAC [12], STARS [13], ASTROS [14], and ZAERO [15] implemented one of the classical LS and MS method algorithms.

Another code called FAMUSS [16] implemented a new method, different from LS and MS methods. In this method, the state-space matrices elements in the aeroservoelastic code FAMUSS [16] were calculated by a *pk* flutter solution and linear and nonlinear LS fits of the direct solution of the system's transfer function frequency response.

A new approximation method based on a Padé approximation gave an approximation error of 12–40 times smaller than the error given by the MS method for the same number of augmented states. This method depended on the choice of the order reduction method [17] and is more expensive in computer time than the MS method.

Cotoi et al. [18] used Chebyshev polynomials and their orthogonality properties to conceive a new approximation method, and excellent results were obtained by this method in comparison with results obtained by the Padé method.

A new method of a combination of *pchip* and *fuzzy clustering techniques* was conceived by Hiliuta et al. [19] for the unsteady force interpolations on a range of nonevenly spaced reduced frequencies. These forces remain in the frequency domain and we will convert them from the frequency into Laplace domain by classical methods such as LS or MS.

We present in this paper a new method based on the mixing of two classical methods LS and the MS. The method presented here gives better results in terms of execution speeds and precision in comparison to the results obtained by the LS method for an F/A-18 aircraft.

II. Mixed Method Presentation

Unsteady generalized aerodynamic forces are approximated in the modified Laplace domain $\bar{s} = jk$ by use of the LS and the MS methods with the following two equations:

$$Q(\bar{s}) = A_0 + A_1\bar{s} + A_2\bar{s}^2 + \sum_{i=1}^{n_{\text{Lags}}} \frac{A_{i+2}}{\bar{s} + b_i} \bar{s} \quad (1)$$

$$Q(\bar{s}) = A_0 + A_1\bar{s} + A_2\bar{s}^2 + D[\bar{s}I - R]^{-1}E\bar{s} \quad (2)$$

where Eq. (1) represents the LS analytical form of the approximated unsteady aerodynamic forces from the frequency domain into Laplace domain and Eq. (2) represents the MS analytical form of these aerodynamic forces. By mixing Eqs. (1) and (2), we obtain the unsteady generalized aerodynamic forces in the modified Laplace domain \bar{s} as follows:

$$Q(\bar{s}) = A_0 + A_1\bar{s} + A_2\bar{s}^2 + \sum_{i=1}^{n_{\text{Lags}}} \frac{A_{i+2}}{\bar{s} + b_i} \bar{s} + D[\bar{s}I - R]^{-1}E\bar{s} \quad (3)$$

where A_0, A_1, A_2, \dots , and A_{i+2} are $(m \times n)$ matrices, D is a

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$(m * n_{Lags})$ matrix, R is a $(n_{Lags} * n_{Lags})$ diagonal matrix, E is a $(n_{Lags} * n)$ matrix, n_{Lags} is the number of lag terms, and b_i are the lag terms (where $i = 1, 2, \dots, n_{Lags}$). We will further write the last term $D[\bar{s}I - R]^{-1}E\bar{s}$ of Eq. (3) under the same analytical LS form of the fourth term

$$\sum_{i=1}^{n_{Lags}} \frac{A_{i+2}}{\bar{s} + b_i} \bar{s}$$

of the same Eq. (3) which becomes

$$Q(\bar{s}) = A_0 + A_1\bar{s} + A_2\bar{s}^2 + \sum_{i=1}^{n_{Lags}=n_{LS}=n_{MS}} \frac{A_{i+2}}{\bar{s} + b_i} \bar{s} \quad (4)$$

where the total number of lag terms calculated by the LS method is denoted by n_{LS} and by the MS method is denoted by n_{MS} . We assume that the lag terms number calculated by both methods is equal and this assumption is written under the form $n_{LS} = n_{MS}$. From this assumption (and further, common denominator), the lag terms calculated by the LS method are equal to the lag terms calculated by the MS method: $b_{iLS} = b_{iMS}$. With these assumptions, the last term of Eq. (4) is written as follows:

$$\sum_{i=1}^{n_{Lags}} \frac{A_{i+2}}{\bar{s} + b_i} = \sum_{i=1}^{n_{LS}} \frac{A_{i+2}}{\bar{s} + b_i} + D[\bar{s}I - R]^{-1}E \quad (5)$$

where for the simplest case of a number of lag terms $n_{LS} = n_{MS} = 2$, from the left-hand side matrices we calculate the right-hand side matrices A_3 and A_4 which are further written:

$$A_3 = \begin{bmatrix} a_{11}^3 & a_{12}^3 & a_{13}^3 & \dots & a_{1m}^3 \\ a_{21}^3 & a_{22}^3 & a_{23}^3 & \dots & a_{2m}^3 \\ a_{31}^3 & a_{32}^3 & a_{33}^3 & \dots & a_{3m}^3 \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1}^3 & a_{n2}^3 & a_{n3}^3 & \dots & a_{nm}^3 \end{bmatrix}; \quad A_4 = \begin{bmatrix} a_{11}^4 & a_{12}^4 & a_{13}^4 & \dots & a_{1m}^4 \\ a_{21}^4 & a_{22}^4 & a_{23}^4 & \dots & a_{2m}^4 \\ a_{31}^4 & a_{32}^4 & a_{33}^4 & \dots & a_{3m}^4 \\ \dots & \dots & \dots & \dots & \dots \\ a_{n1}^4 & a_{n2}^4 & a_{n3}^4 & \dots & a_{nm}^4 \end{bmatrix} \quad (6)$$

where exponents 3 and 4 correspond to the A_3 and A_4 matrices elements and are denoted by a_{ij}^3 and a_{ij}^4 . In the A_3 matrix, we write the coefficients $a_{2j}^3, a_{3j}^3, \dots, a_{nj}^3$ as a function of the first row coefficients expressed as a_{11j}^3 , where $j = 1, 2, \dots, m$ as shown:

$$A_3 = \begin{bmatrix} a_{11}^3 * (1) & a_{12}^3 * (1) & a_{13}^3 * (1) & \dots & a_{1m}^3 * (1) \\ a_{11}^3 * (\alpha_2^3) + r_{21}^3 & a_{12}^3 * (\alpha_2^3) + r_{22}^3 & a_{13}^3 * (\alpha_2^3) + r_{23}^3 & \dots & a_{1m}^3 * (\alpha_2^3) + r_{2m}^3 \\ a_{11}^3 * (\alpha_3^3) + r_{31}^3 & a_{12}^3 * (\alpha_3^3) + r_{32}^3 & a_{13}^3 * (\alpha_3^3) + r_{33}^3 & \dots & a_{1m}^3 * (\alpha_3^3) + r_{3m}^3 \\ \dots & \dots & \dots & \dots & \dots \\ a_{11}^3 * (\alpha_n^3) + r_{n1}^3 & a_{12}^3 * (\alpha_n^3) + r_{n2}^3 & a_{13}^3 * (\alpha_n^3) + r_{n3}^3 & \dots & a_{1m}^3 * (\alpha_n^3) + r_{nm}^3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \alpha_2^3 & 0 \\ \alpha_3^3 & 0 \\ \dots & \dots \\ \alpha_n^3 & 0 \end{bmatrix} \begin{bmatrix} a_{11}^3 & a_{12}^3 & a_{13}^3 & \dots & a_{1m}^3 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (7)$$

$$+ \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^3 & r_{22}^3 & r_{23}^3 & \dots & r_{2m}^3 \\ r_{31}^3 & r_{32}^3 & r_{33}^3 & \dots & r_{3m}^3 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^3 & r_{n2}^3 & r_{n3}^3 & \dots & r_{nm}^3 \end{bmatrix}$$

where the A_3 matrix is written as a sum of matrices of $(n * m)$ dimensions. The first term in Eq. (7) is a product of two matrices, in which the first matrix of $(n * n)$ dimensions has only its first column nonzero and the second matrix has its first row also nonzero. In Eq. (7), the elements $\alpha_2^3, \alpha_3^3, \dots, \alpha_n^3$ of the first matrix are real factors arbitrarily chosen and the elements r_{ij}^3 of the third matrix are real residual values.

The A_4 matrix is further written under a similar form as the A_3 matrix, the only difference between their notation is found in the notation of the A_4 matrix elements (exponent 4 is used instead of exponent 3):

$$A_4 = \begin{bmatrix} 1 & 0 \\ \alpha_2^4 & 0 \\ \alpha_3^4 & 0 \\ \dots & \dots \\ \alpha_n^4 & 0 \end{bmatrix} \begin{bmatrix} a_{11}^4 & a_{12}^4 & a_{13}^4 & \dots & a_{1m}^4 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^4 & r_{22}^4 & r_{23}^4 & \dots & r_{2m}^4 \\ r_{31}^4 & r_{32}^4 & r_{33}^4 & \dots & r_{3m}^4 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^4 & r_{n2}^4 & r_{n3}^4 & \dots & r_{nm}^4 \end{bmatrix} \quad (8)$$

The coefficients in the first column $[1, \alpha_2^4, \alpha_3^4, \dots, \alpha_n^4]$ and in the first row $[a_{12}^4, a_{13}^4, \dots, a_{1m}^4]$ of the A_4 matrix given by Eq. (8) are replaced in the last column and the last row, as follows:

$$A_4 = \begin{bmatrix} 0 & 1 \\ 0 & \alpha_2^4 \\ 0 & \alpha_3^4 \\ \dots & \dots \\ 0 & \alpha_n^4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ a_{11}^4 & a_{12}^4 & a_{13}^4 & \dots & a_{1m}^4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^4 & r_{22}^4 & r_{23}^4 & \dots & r_{2m}^4 \\ r_{31}^4 & r_{32}^4 & r_{33}^4 & \dots & r_{3m}^4 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^4 & r_{n2}^4 & r_{n3}^4 & \dots & r_{nm}^4 \end{bmatrix} \quad (9)$$

The A_3 and A_4 matrices expressed by Eqs. (7–9) are written under the generalized A_{i+2} matrix form ($i = 1, 2, \dots, n_{LS}$) where the $i + 2$ column and $i + 2$ row positions are changed into the i column and i row positions.

For a number of lag terms $n_{Lags} = 2$, the following sum is computed:

$$\sum_{i=1}^{n_{Lags}} \frac{A_{i+2}}{\bar{s} + b_i} = \frac{A_3}{\bar{s} - b_1} + \frac{A_4}{\bar{s} - b_2} \quad (10)$$

which becomes, after replacing A_3 and A_4 matrices given by Eqs. (7) and (9) into Eq. (10):

$$\begin{aligned} \frac{A_3}{\bar{s}-b_1} + \frac{A_4}{\bar{s}-b_2} = \frac{1}{\bar{s}-b_1} & \begin{bmatrix} 1 & 0 \\ \alpha_2^3 & 0 \\ \alpha_3^3 & 0 \\ \dots & \dots \\ \alpha_n^3 & 0 \end{bmatrix} \begin{bmatrix} a_{11}^3 & a_{12}^3 & a_{13}^3 & \dots & a_{1m}^3 \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^3 & r_{22}^3 & r_{23}^3 & \dots & r_{2m}^3 \\ r_{31}^3 & r_{32}^3 & r_{33}^3 & \dots & r_{3m}^3 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^3 & r_{n2}^3 & r_{n3}^3 & \dots & r_{nm}^3 \end{bmatrix} \\ + \frac{1}{\bar{s}-b_2} & \begin{bmatrix} 0 & 1 \\ 0 & \alpha_2^4 \\ 0 & \alpha_3^4 \\ \dots & \dots \\ 0 & \alpha_n^4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ a_{11}^4 & a_{12}^4 & a_{13}^4 & \dots & a_{1m}^4 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^4 & r_{22}^4 & r_{23}^4 & \dots & r_{2m}^4 \\ r_{31}^4 & r_{32}^4 & r_{33}^4 & \dots & r_{3m}^4 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^4 & r_{n2}^4 & r_{n3}^4 & \dots & r_{nm}^4 \end{bmatrix} \end{aligned} \quad (11)$$

Equation (11) is written under the following condensed form:

$$\begin{aligned} \frac{A_3}{\bar{s}-b_1} + \frac{A_4}{\bar{s}-b_2} = & \begin{bmatrix} 1 & 1 \\ \alpha_2^3 & \alpha_2^4 \\ \alpha_3^3 & \alpha_3^4 \\ \dots & \dots \\ \alpha_n^3 & \alpha_n^4 \end{bmatrix} \begin{bmatrix} \frac{1}{\bar{s}-b_1} & 0 \\ 0 & \frac{1}{\bar{s}-b_2} \end{bmatrix} \begin{bmatrix} a_{11}^3 & a_{12}^3 & a_{13}^3 & \dots & a_{1m}^3 \\ a_{11}^4 & a_{12}^4 & a_{13}^4 & \dots & a_{1m}^4 \end{bmatrix} + \frac{1}{\bar{s}-b_1} \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^3 & r_{22}^3 & r_{23}^3 & \dots & r_{2m}^3 \\ r_{31}^3 & r_{32}^3 & r_{33}^3 & \dots & r_{3m}^3 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^3 & r_{n2}^3 & r_{n3}^3 & \dots & r_{nm}^3 \end{bmatrix} \\ + \frac{1}{\bar{s}-b_2} & \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^4 & r_{22}^4 & r_{23}^4 & \dots & r_{2m}^4 \\ r_{31}^4 & r_{32}^4 & r_{33}^4 & \dots & r_{3m}^4 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^4 & r_{n2}^4 & r_{n3}^4 & \dots & r_{nm}^4 \end{bmatrix} \end{aligned} \quad (12)$$

The unknown matrices A_3, A_4, D , and E are further calculated by terms identification and we obtain

$$\begin{aligned} A_3 = & \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^3 & r_{22}^3 & r_{23}^3 & \dots & r_{2m}^3 \\ r_{31}^3 & r_{32}^3 & r_{33}^3 & \dots & r_{3m}^3 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^3 & r_{n2}^3 & r_{n3}^3 & \dots & r_{nm}^3 \end{bmatrix}, \quad A_4 = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ r_{21}^4 & r_{22}^4 & r_{23}^4 & \dots & r_{2m}^4 \\ r_{31}^4 & r_{32}^4 & r_{33}^4 & \dots & r_{3m}^4 \\ \dots & \dots & \dots & \dots & \dots \\ r_{n1}^4 & r_{n2}^4 & r_{n3}^4 & \dots & r_{nm}^4 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 1 \\ \alpha_2^3 & \alpha_2^4 \\ \alpha_3^3 & \alpha_3^4 \\ \dots & \dots \\ \alpha_n^3 & \alpha_n^4 \end{bmatrix} \\ [sI - R]^{-1} = & \begin{bmatrix} \frac{1}{s-b_1} & 0 \\ 0 & \frac{1}{s-b_2} \end{bmatrix} \quad \text{and} \quad E = \begin{bmatrix} a_{11}^3 & a_{12}^3 & a_{13}^3 & \dots & a_{1m}^3 \\ a_{11}^4 & a_{12}^4 & a_{13}^4 & \dots & a_{1m}^4 \end{bmatrix} \end{aligned} \quad (13)$$

From Eq. (13), it is seen that elements of the first rows of the A_{i+2}^{LS} matrices are equal to zero and the elements of the first rows of the D matrix are equal to 1. The inverse problem requires very few manipulations of matrices, no iterative solutions are necessary, and therefore, the computing time is small.

One advantage of the formulation described in this method is that it is simple, as shown in Eq. (3). One particular case is the one in which the residual values are equal to zero so that we can write $r_{ij}^k = 0; i = 1, 2, \dots, n; j = 1, 2, \dots, m; k = 1, 2, \dots, n_{Lag}$. In this case, Eq. (12) may be written as follows:

$$\frac{A_3}{\bar{s}-b_1} + \frac{A_4}{\bar{s}-b_2} + \dots + \frac{A_{k+2}}{\bar{s}-b_k} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \alpha_2^3 & \alpha_2^4 & \dots & \alpha_2^{k+2} \\ \alpha_3^3 & \alpha_3^4 & \dots & \alpha_3^{k+2} \\ \dots & \dots & \dots & \dots \\ \alpha_n^3 & \alpha_n^4 & \dots & \alpha_n^{k+2} \end{bmatrix} \begin{bmatrix} \frac{1}{\bar{s}-b_1} & 0 & \dots & 0 \\ 0 & \frac{1}{\bar{s}-b_2} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \frac{1}{\bar{s}-b_k} \end{bmatrix} \begin{bmatrix} a_{11}^3 & a_{12}^3 & \dots & a_{1m}^3 \\ a_{11}^4 & a_{12}^4 & \dots & a_{1m}^4 \\ \dots & \dots & \dots & \dots \\ a_{11}^{k+2} & a_{12}^{k+2} & \dots & a_{1m}^{k+2} \end{bmatrix} \quad (14)$$

The A_3 and A_4 matrices are denoted by A_{i+2} , where $i = 1$ and 2 . We first assume that the residual elements are zero, then the A_{i+2} matrix is written under the following form:

$$A_{i+2} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ \alpha_2^{i+2} & 0 & \dots & 0 \\ \alpha_3^{i+2} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \alpha_n^{i+2} & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} a_{11}^{i+2} & a_{12}^{i+2} & a_{13}^{i+2} & \dots & a_{1m}^{i+2} \\ 0 & 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (15)$$

We secondly replace all zeros with negligible very small values, denoted here by epsilon (eps^+ and eps^*), and matrix A_{i+2} given by Eq. (15) is written with these notations as follows:

$$A_{i+2} = \begin{bmatrix} 1 & eps_{12}^+ & \dots & eps_{1n_{Lag}}^+ \\ \alpha_2^{i+2} & eps_{22}^+ & \dots & eps_{2n_{Lag}}^+ \\ \alpha_3^{i+2} & eps_{32}^+ & \dots & eps_{3n_{Lag}}^+ \\ \dots & \dots & \dots & \dots \\ \alpha_n^{i+2} & eps_{n2}^+ & \dots & eps_{nn_{Lag}}^+ \end{bmatrix} \begin{bmatrix} a_{11}^{i+2} & a_{12}^{i+2} & a_{13}^{i+2} & \dots & a_{1m}^{i+2} \\ eps_{21}^* & eps_{22}^* & eps_{23}^* & \dots & eps_{2m}^* \\ \dots & \dots & \dots & \dots & \dots \\ eps_{n_{Lag}1}^* & eps_{n_{Lag}2}^* & eps_{n_{Lag}3}^* & \dots & eps_{n_{Lag}m}^* \end{bmatrix} \quad (16)$$

III. Results

Using the approximations by our mixed state method and by the LS method of the aerodynamic forces from the frequency into Laplace domain and by their integration in the pk flutter method, several relative error flutter speeds and frequency results are obtained. Comparison of these results is given in this section.

For this comparison, we introduce the following notations for the flutter equivalent airspeeds EAS and frequencies f calculated by use of the pk method: EAS_{LS} is the equivalent airspeed calculated with aerodynamic forces approximated in the Laplace domain by the LS method, EAS_{MxS} is the equivalent airspeed calculated with aerodynamic forces approximated in the Laplace domain by our method, f_{LS} is the frequency calculated with aerodynamic forces approximated in the Laplace domain by the LS method, and f_{MxS} is the frequency calculated with aerodynamic forces approximated in the Laplace domain with our method. The relative errors corresponding to EAS and f calculated by the LS and MxState methods with respect to EAS_{pk} and f_{pk} calculated with the pk method are defined by the following equations:

$$Err_{EAS_{LS}} = 100 * \text{abs}\left(\frac{EAS_{LS} - EAS_{pk}}{EAS_{pk}}\right) \quad (17a)$$

$$Err_{EAS_{MxS}} = 100 * \text{abs}\left(\frac{EAS_{MxS} - EAS_{pk}}{EAS_{pk}}\right) \quad (17b)$$

$$Err_{f_{LS}} = 100 * \text{abs}\left(\frac{f_{LS} - f_{pk}}{f_{pk}}\right) \quad (17c)$$

$$Err_{f_{MxS}} = 100 * \text{abs}\left(\frac{f_{MxS} - f_{pk}}{f_{pk}}\right) \quad (17d)$$

The following four cases are presented in this section (for two Mach numbers $M = 1.4$ and 1.6 and for two types of modes: symmetric and antisymmetric):

- 1) Case 1: Mach number $M = 1.4$, F/A-18 symmetric modes;
- 2) Case 2: Mach number $M = 1.4$, F/A-18 antisymmetric modes;
- 3) Case 3: Mach number $M = 1.6$, F/A-18 symmetric modes;
- 4) Case 4: Mach number $M = 1.6$, F/A-18 antisymmetric modes.

Figures 1, 3, 5, and 7 represent, for cases 1–4, in the form of 3-D bars, and for 1–7 lag terms, the relative errors of flutter speeds calculated with the LS method by use of Eq. (17a) and with our method by use of Eq. (17b). Figures 2, 4, 6, and 8 represent in the visual form of 3-D bars, for cases 1–4, the relative errors of flutter frequencies calculated by the LS and the mixed states method for 1–7 lags, by use of Eqs. (17c) and (17d).

Tables 1–4 represent the numerical relative errors of flutter speeds and frequency results by the LS and the mixed state methods for 1–7 lag terms shown already in the 3-D bar form in Figs. 1–8.

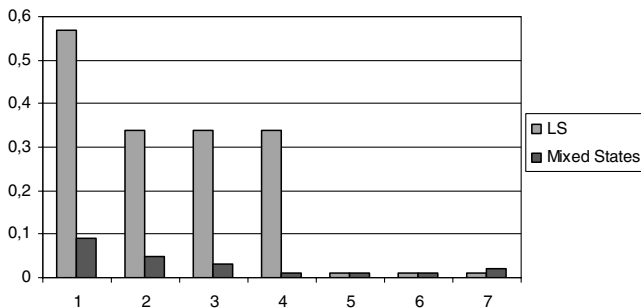


Fig. 1 Visual representation of relative errors of flutter speeds calculated by the LS and mixed state methods versus the set of 1–7 lag terms for case 1: $M = 1.4$, symmetric modes.

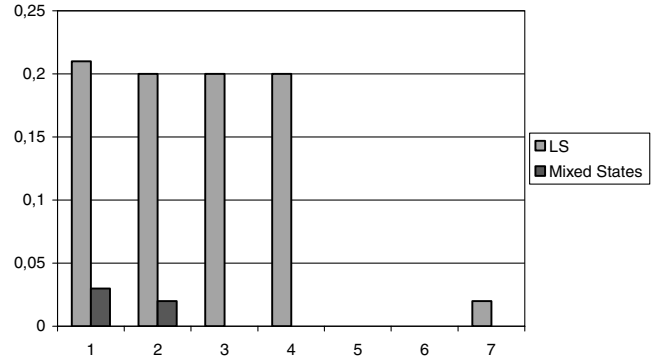


Fig. 2 Visual representation of relative errors of flutter frequencies calculated by the LS and mixed state methods versus the set of 1–7 lag terms for case 1: $M = 1.4$, symmetric modes.

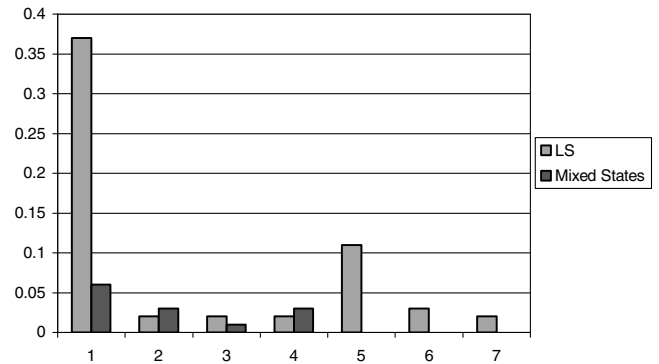


Fig. 3 Visual representation of relative errors of flutter speeds calculated by the LS and mixed state methods versus the set of 1–7 lag terms for case 2: $M = 1.4$, antisymmetric modes.

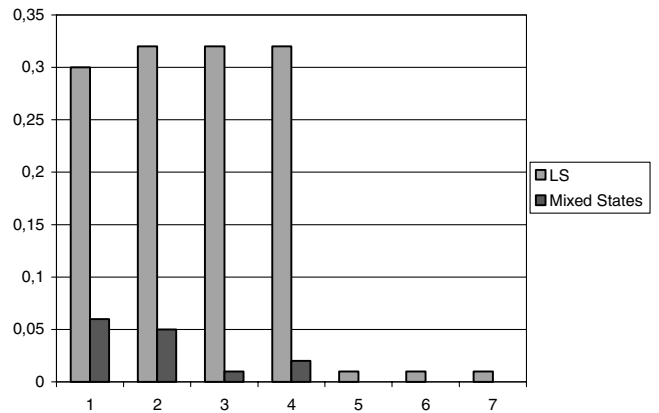


Fig. 4 Visual representation of relative errors of flutter frequencies calculated by the LS and mixed state methods versus the set of 1–7 lag terms for case 2: $M = 1.4$, antisymmetric modes.

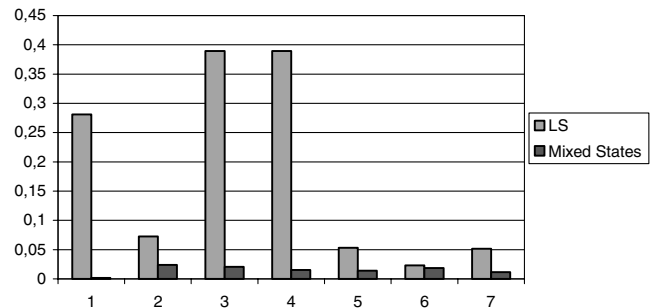


Fig. 5 Visual representation of relative errors of flutter speeds calculated by the LS and mixed state methods versus the set of 1–7 lag terms for case 3: $M = 1.6$, symmetric modes.

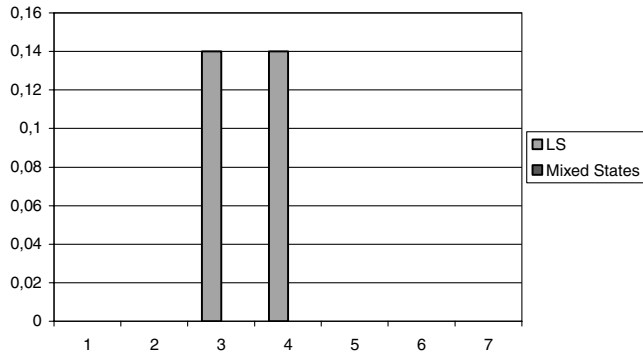


Fig. 6 Visual representation of relative errors of flutter frequencies calculated by the LS and mixed state methods versus the set of 1–7 lag terms for case 3: $M = 1.6$, symmetric modes.

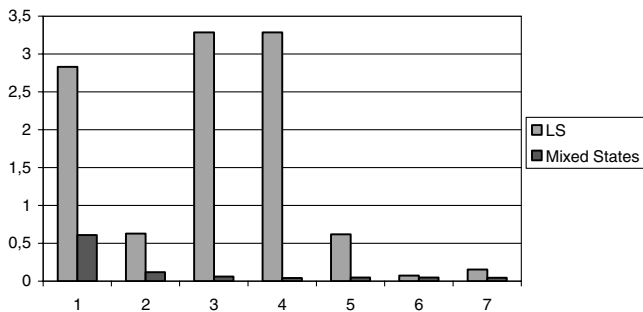


Fig. 7 Visual representation of relative errors of flutter speeds calculated by the LS and mixed state methods versus the set of 1–7 lag terms for case 4: $M = 1.6$, antisymmetric modes.

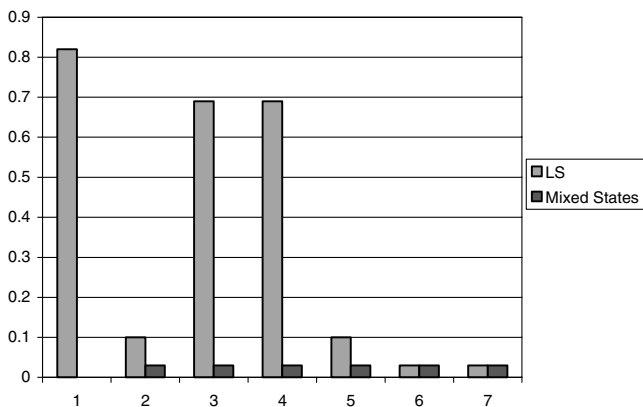


Fig. 8 Visual representation of relative errors of flutter frequencies calculated by the LS and mixed state methods versus the set of 1–7 lag terms for case 4: $M = 1.6$, antisymmetric modes.

In other words, flutter results expressed numerically in Table 1 are visually represented in Figs. 1 and 2, results expressed in Table 2 are represented in Figs. 3 and 4, and results expressed in Tables 3 and 4 are represented in Figs. 5 and 6, and in Figs. 7 and 8, respectively.

To compare the flutter results by these two methods, we will first find in Tables 1–4 corresponding to cases 1–4 the smallest numerical values for the equivalent airspeeds and frequencies calculated by the LS method (second and third columns in these tables). Once this value is determined, we will look for the closest value to this value calculated by the mixed state method (fourth and fifth columns of these tables). The same evaluation criteria will be used for the flutter frequencies results, to find the optimum number of lag terms.

For case 1, in Table 1, by the LS method, the smallest first flutter speed error is 0.01% and the smallest flutter frequency error is 0%; both values occur for a number of five lag terms. The closest first

Table 1 Numerical representation of relative errors of flutter speeds and frequencies calculated by the LS and mixed state methods corresponding to a set of 1–7 lag terms for case 1: $M = 1.4$, symmetric modes

No. of lag terms	LS method		Mixed state method	
	$EAS_{LS}, \%$	$f_{LS}, \%$	$EAS_{MxS}, \%$	$f_{MxS}, \%$
1	0.57	0.21	0.09	0.03
2	0.34	0.20	0.05	0.02
3	0.34	0.20	0.03	0.00
4	0.34	0.20	0.01	0.00
5	0.01	0.00	0.01	0.00
6	0.01	0.00	0.01	0.00
7	0.01	0.02	0.02	0.00

Table 2 Numerical representation of relative errors of flutter speeds and frequencies calculated by the LS and mixed state methods corresponding to a set of 1–7 lag terms for case 2: $M = 1.4$, antisymmetric modes

No. of lag terms	LS method		Mixed state method	
	$EAS_{LS}, \%$	$f_{LS}, \%$	$EAS_{MxS}, \%$	$f_{MxS}, \%$
1	0.37	0.30	0.06	0.06
2	0.02	0.32	0.03	0.05
3	0.02	0.32	0.01	0.01
4	0.02	0.32	0.03	0.02
5	0.11	0.01	0.00	0.00
6	0.03	0.01	0.00	0.00
7	0.02	0.01	0.00	0.00

Table 3 Numerical representation of relative errors of flutter speeds and frequencies calculated by the LS and mixed state methods corresponding to a set of 1–7 lag terms for case 3: $M = 1.6$, symmetric modes

No. of lag terms	LS method		Mixed state method	
	$EAS_{LS}, \%$	$f_{LS}, \%$	$EAS_{MxS}, \%$	$f_{MxS}, \%$
1	0.28	0	0.00	0
2	0.07	0	0.02	0
3	0.39	0.14	0.02	0
4	0.39	0.14	0.02	0
5	0.05	0	0.01	0
6	0.02	0	0.02	0
7	0.05	0	0.01	0

Table 4 Numerical representation of relative errors of flutter speeds and frequencies calculated by the LS and mixed state methods corresponding to a set of 1–7 lag terms for case 4: $M = 1.6$, antisymmetric modes

No. of lag terms	LS method		Mixed state method	
	$EAS_{LS}, \%$	$f_{LS}, \%$	$EAS_{MxS}, \%$	$f_{MxS}, \%$
1	2.83	0.82	0.61	0
2	0.63	0.10	0.12	0.03
3	3.29	0.69	0.06	0.03
4	3.29	0.69	0.04	0.03
5	0.62	0.10	0.05	0.03
6	0.07	0.03	0.05	0.03
7	0.16	0.03	0.05	0.03

Table 5 Number of optimal lag terms calculated for the LS and mixed state methods

N_{lags} for EAS_{LS}	LS method		Mixed state method	
	N_{lags} for f_{LS}	N_{lags} for $EAS_{MxState}$	N_{lags} for f_{MxS}	N_{lags} for $f_{MxState}$
5	5	4	3	
2	5	2	3	
6	1	2	1	
6	6	3	2	

flutter speeds and frequency errors by the mixed state method occur for a number of four lag terms (for the flutter speeds) and for a number of three lag terms (for flutter frequencies).

By use of the same criteria and reasoning in Tables 2–4, for cases 2–4, we found results summarized in Table 5, from where we found that the smaller number of lag terms are used in the mixed state method in comparison to the number of lag terms used in the classical LS method and therefore, convergence time is smaller in case of our method use with respect to the LS method.

IV. Conclusions

This new mixed (LS and MS) formulation called the MxState method will allow us to obtain the MS approximation without passing through a long iterative algorithm. In this manner we minimize the number of lag terms in the MS approximation term, which means that the lags applied to the two terms:

$$\sum_{i=1}^{n_{\text{Lag}}} \frac{A_{i+2}}{s + b_i} s$$

and $D[sI - R]^{-1}Es$ will be the same. We compared the flutter speeds and frequencies found by the flutter standard nonlinear method with the flutter speeds and frequencies found by the LS method, and with the flutter speeds and frequencies found by the new mixed state MxState for an F/A-18 aircraft.

From Table 5, we found that the MxState method with a smaller number of lag terms gives results that are very close to the results of the standard pk flutter and the LS approximation method with a higher number of lag terms. Results were presented on an F/A-18 aircraft with 14 symmetric modes and 14 antisymmetric modes.

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