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Method Based on Chebyshev Polynomials for Aeroservoelastic Interactions on an F/A-18 Aircraft

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I. Introduction

AEROSERVOELASTICITY studies on a fly-by-wire aircraft involve studies of three main disciplines: unsteady aerodynamics, elastic aircraft structure, and controls. In aeroelasticity, unsteady aerodynamic forces are calculated in the frequency values for a range of reduced frequency values by use of methods implemented in aeroelastic analysis software such as NASTRAN [1], and are converted into the Laplace domain (for aeroservoelasticity studies) by use of the following methods [2,3]: least squares (LS), matrix Padé (MP), and minimum state (MS).

An interpolation method is described that uses Chebyshev polynomials and their properties, programmed by use of existing Matlab and Maple routines. The *pk*–flutter method was developed in Matlab, and details of this method and the flutter results obtained on the aircraft test model (ATM) in STARS were published [4]. The method of approximation by Chebyshev polynomials has been applied successfully for open-loop analysis [5] and for closed-loop analysis [6] on the aircraft test model. In this paper, the Chebyshev method was applied to the F/A-18 aircraft.

II. Aircraft Equations of Motion in the Laplace Domain

The equations of motion of the aircraft are written as follows:

$$[\tilde{\boldsymbol{M}}\bar{s}^2 + \tilde{\boldsymbol{C}}\,\bar{s} + \tilde{\boldsymbol{K}}]\eta(\bar{s}) + q_{\text{dyn}}\boldsymbol{Q}(\bar{s})\eta(\bar{s}) = 0 \tag{1}$$

where the generalized mass $\tilde{\boldsymbol{M}}$, stiffness $\tilde{\boldsymbol{K}}$, and damping $\tilde{\boldsymbol{C}}$ are defined as

$$\tilde{\boldsymbol{M}} = \boldsymbol{\Phi}^T\boldsymbol{M}\boldsymbol{\Phi}, \qquad \tilde{\boldsymbol{C}} = \boldsymbol{\Phi}^T\boldsymbol{C}\boldsymbol{\Phi}, \qquad \tilde{\boldsymbol{K}} = \boldsymbol{\Phi}^T\boldsymbol{K}\boldsymbol{\Phi} \tag{2}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ are the modal mass, damping, and stiffness matrices, and $\boldsymbol{\Phi}$ is the modal matrix. In Eq. (1), $\boldsymbol{Q}(\bar{s})$ is the Laplace transform of the aerodynamic forces in the frequency domain $\boldsymbol{Q}(k, M) = \boldsymbol{\Phi}^T \mathrm{A}_e(k)\boldsymbol{\Phi}$, where $\mathrm{A}_e(k)$ are the aerodynamic influence coefficients dependent on the reduced frequency range $k = \omega b/V$, where ω is the natural frequency, b is the wing semichord length, and V is the true airspeed.

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III. Chebyshev Theory

Chebyshev polynomials of the first kind* are a set of orthogonal polynomials defined as the Chebyshev differential equation solutions. These polynomials are calculated by use of trigonometric multiple-angle equations.

Any continuous function $f(x)$ may be expressed using Chebyshev polynomials as follows:

$$f(x) = \frac{1}{2}c_0 + \sum_{j=1}^{\infty} c_j T_j(x) \tag{3}$$

In Eq. (3), the coefficients c_j are expressed as follows:

$$c_j = \frac{2}{\pi}\int_{-1}^{1} \frac{f(x)T_j(x)}{\sqrt{1-x^2}}\,\mathrm{d}x; \qquad j = 1, 2, \ldots \tag{4}$$

and the Chebyshev polynomials $T_j(x)$ are given by

$$T_j(x) = \cos[j \arccos(x)] \tag{5}$$

Chebyshev polynomials are used in engineering applications because of their orthogonality properties, which provide a calculated predetermined bandwidth value for the error of approximation. This condition is written in an integral form:

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} T_r(x)T_s(x)\,\mathrm{d}x = \begin{cases} 0, & r \neq s \\ \pi, & r = s = 0 \\ \frac{\pi}{2}, & r = s \neq 0 \end{cases} \tag{6}$$

where $T_r(x)$ and $T_s(x)$ are Chebyshev polynomials of ranks r and s, respectively. The recurrence relationships for Chebyshev polynomials may be written as

$$\begin{cases} T_0(x) = 1 \\ T_1(x) = x \\ T_{r+1}(x) = 2xT_r(x) - T_{r-1}(x) \end{cases} \tag{7}$$

From Eq. (5), by replacing $r = j$, the condition required to find the Chebyshev polynomials solution is imposed:

$$T_r(x) = \cos[r \arccos(x)] = 0 \tag{8}$$

In Eq. (8), the following variable change is made:

$$y = r \arccos(x) \tag{9}$$

from where

$$\cos(y) = 0 \tag{10}$$

which gives the solution

$$y = (2j+1)\frac{\pi}{2}; \qquad j = 0, 1, \ldots \tag{11}$$

We further replace y given by Eq. (9) into Eq. (11), and the following equation is obtained:

*Data available online at http://mathworld.wolfram.com/ChebyshevPolynomialoftheFirstKind.html [cited 15 November 2006].

$$r \arccos(x) = (2j+1)\frac{\pi}{2} \Rightarrow \arccos(x) = \frac{(2j+1)\pi}{2r} \tag{12}$$

which can be written in the following form:

$$x = \cos\frac{(2j+1)\pi}{2r}; \qquad j = 0, 1, \ldots, r-1 \tag{13}$$

$T_r(x)$ is a function defined by cosines, which lets us conclude that between two solutions of this function we will find an extreme of $|1|$ amplitude exactly in the middle of the interval, specifically at

$$x = \cos\frac{j\pi}{r}; \qquad j = 0, 1, \ldots, r \tag{14}$$

IV. Description of the New Method Based on Chebyshev Theory

The Chebyshev approximation method provides an excellent approximation compared to the Padé method. However, because the Chebyshev polynomials have to be generated using the data provided for the F/A-18 aircraft, a process that manipulates some very large differences between the values of elements contained in the unsteady generalized aerodynamic force matrices (1e + 10), some restraints regarding the threshold of the approximation error had to be imposed; i.e., for smaller elements we have imposed a maximum error value of 1e − 4, and for larger elements a maximum error value of 1e − 3. Without these restraints, the Chebyshev polynomials cannot be generated.

A power series development was realized by use of the "*chebyshev*" function that is a part of Maple's kernel in Matlab for each element (ij) of the unsteady aerodynamic forces denoted here by $\boldsymbol{Q}_{ij}$, as follows:

$$\boldsymbol{Q}_{ij}(s) = \frac{1}{2}c_0^{(ij)} + \sum_{n=1}^{\infty} c_n^{(ij)} T_n^{(ij)}(s) \tag{15}$$

where

$$c_n^{(ij)} = \frac{2}{\pi}\int_{-1}^{1} \frac{\boldsymbol{Q}_{ij}(s)T_n^{(ij)}(s)}{\sqrt{(1-s^2)}}\,\mathrm{d}s; \qquad n = 0, 1, \ldots$$

Next, a second approximation for these unsteady forces was realized using the "*chebpade*" function preprogrammed in Maple's kernel in Matlab, represented by a ratio of rational fractions:

$$\hat{\boldsymbol{Q}}_{ij}(s) = \frac{\sum_{n=0}^{M} a_n^{(ij)} T_n^{(ij)}(s)}{1 + \sum_{n=1}^{P} b_n^{(ij)} T_n^{(ij)}(s)} \tag{16}$$

where $M = P + 2$.

The Chebyshev polynomial orthogonality properties are integrated in Eq. (16). Different values for the degree of the denominator, and therefore for the numerator, are considered in the presentation of our results. For comparison purposes, we next consider the Padé classical method, which gives a polynomial fractional form for the unsteady force representation in the Laplace domain and, therefore, will reduce the order of the system. The results obtained by the two methods (Chebyshev and Padé) are given in the next section in the form of total normalized error of the unsteady aerodynamic force approximations.

V. Presentation of Results Obtained by Chebyshev and Padé Methods

The unsteady generalized aerodynamic forces, acting on an F/A-18 aircraft for a range of 14 reduced frequencies $k = 0.01$, 0.1, 0.2, 0.3, 0.4, 0.5, 0.588, 0.625, 0.67, 0.71, 0.77, 0.83, 0.91, and 1, and Mach numbers $M = 1.4$ and 1.6, are calculated. The elements of these matrices are denoted by $\boldsymbol{Q}(i, j)$ where i denotes the index of the row number and j represents the index of the column number.

In Fig. 1, total normalized approximation errors of unsteady aerodynamic forces by Padé and Chebyshev theories are shown for their real parts, and we notice that a smaller approximation error is given by the Chebyshev method compared to that given by the Padé method. The largest differences between these approximation error values are best viewed on this figure at the beginning and at the end of the reduced frequency k interval, because Chebyshev polynomials provide a very good approximation even at the end points of the reduced frequency interval.

An approximation order $[M, P] = [6, 4]$ gives the numerator maximum rank $M = 6$ and the denominator maximum rank $P = 4$ in the preceding Eq. (16). For both Padé and Chebyshev polynomials the approximation order is defined in a similar manner and the same approximation order was considered in Fig. 1.

Because of Chebyshev polynomials properties, we were able to impose a bandwidth for the error convergence on the approximation for each element of the unsteady generalized aerodynamic force matrices.

We normalize the approximation error J for each element of the $\boldsymbol{Q}$ aerodynamic unsteady force matrix (the real and the imaginary part), for each reduced frequency k by use of the following two equations for the real and the imaginary part, respectively:

$$J_{Q_{\mathrm{real}}} = \sum_{k=1}^{14}\left[\sum_{i=1}^{N_{\mathrm{modes}}}\left(\sum_{j=1}^{N_{\mathrm{modes}}} \frac{|\boldsymbol{Q}_{ij\boldsymbol{R}\mathrm{new}} - \boldsymbol{Q}_{ij\boldsymbol{R}\mathrm{old}}|}{\sqrt{|\boldsymbol{Q}_{ij}|^2}}\right)\right] \times 100\%$$
$$J_{Q_{\mathrm{imag}}} = \sum_{k=1}^{14}\left[\sum_{i=1}^{N_{\mathrm{modes}}}\left(\sum_{j=1}^{N_{\mathrm{modes}}} \frac{|\boldsymbol{Q}_{ij\boldsymbol{I}\mathrm{new}} - \boldsymbol{Q}_{ij\boldsymbol{I}\mathrm{old}}|}{\sqrt{|\boldsymbol{Q}_{ij}|^2}}\right)\right] \times 100\% \tag{17}$$

In these equations, $\boldsymbol{Q}_{\boldsymbol{R}\mathrm{old}}$ and $\boldsymbol{Q}_{\boldsymbol{I}\mathrm{old}}$ are the real and the imaginary parts of the unsteady aerodynamic forces calculated first (this is the reason why "old" is added at the index of the aerodynamic forces) in the frequency domain, and $\boldsymbol{Q}_{\boldsymbol{R}\mathrm{new}}$ and $\boldsymbol{Q}_{\boldsymbol{I}\mathrm{new}}$ are the real and the imaginary parts of the unsteady aerodynamic forces by use of Chebyshev or Padé methods. N_{modes} is the total number of modes (the $\boldsymbol{Q}$ matrix rows or column numbers that are equal), k is the index of the reduced frequency (for a total of 14 reduced frequencies, values noted at the beginning of this section), and J is the total normalized error.

Figure 1 shows results for the F/A-18 aircraft symmetric modes at Mach number $M = 1.4$. Results for the F/A-18 aircraft symmetric

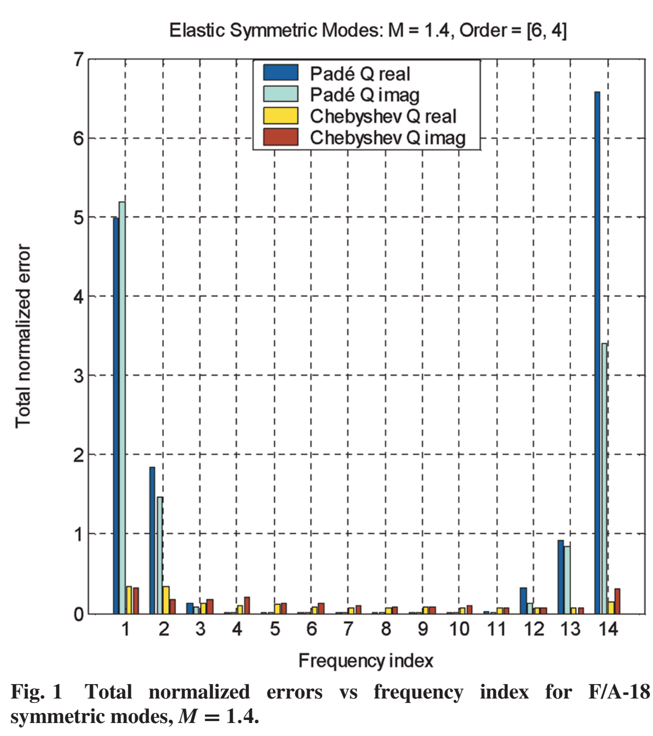

Fig. 1 Total normalized errors vs frequency index for F/A-18 symmetric modes, $M = 1.4$.

Table 1 Total normalized errors

		M = 1.4			M = 1.6		
		[4, 2]	[5, 3]	[6, 4]	[4, 2]	[5, 3]	[6, 4]
Elastic symmetric modes	$J_{Q_{real}}$ Padé	19.55	57.48	14.80	10.21	27.84	4.14
	$J_{Q_{imag}}$ Padé	17.06	83.79	11.16	8.31	31.04	5.42
	$J_{Q_{real}}$ Chebyshev	5.59	2.48	1.71	3	1.12	0.86
	$J_{Q_{imag}}$ Chebyshev	4.14	2.23	1.99	1.82	1.09	0.57
Elastic antisymmetric modes	$J_{Q_{real}}$ Padé	5.46	13.55	2.66	3.27	7.68	2.21
	$J_{Q_{imag}}$ Padé	5.09	26.46	2.82	2.94	6.34	1.12
	$J_{Q_{real}}$ Chebyshev	2.34	1.14	0.65	1.61	4.77	1.11
	$J_{Q_{imag}}$ Chebyshev	1.90	1.13	0.79	1.62	0.81	0.76

Table 2 Flutter analysis values

		M = 1.4		M = 1.6	
Modes type	Method	Flutter speed, ft/s	Flutter frequency, Hz	Flutter speed, ft/s	Flutter frequency, Hz
Symmetric modes	*pk* Standard	1011.06	28.05	1127.46	28.89
	pk Chebyshev [4, 2]	1011.14	28.05	1127.31	28.89
	pk Chebyshev [5, 3]	1010.99	28.05	1127.57	28.89
	pk Chebyshev [6, 4]	1011.06	28.05	1127.42	28.8908
Antisymmetric modes	*pk* Standard	1045.86	29.1381	1229.7424	30.5467
	pk Chebyshev [4, 2]	1045.00	29.1322	1229.3621	30.5538
	pk Chebyshev [5, 3]	1045.73	29.1370	1229.5796	30.5480
	pk Chebyshev [6, 4]	1045.80	29.1381	1229.2617	30.5457

Table 3 Flutter results (speeds and frequencies) approximation errors

		M = 1.4		M = 1.6	
Modes type	Method	Flutter speed error, %	Flutter frequency error, %	Flutter speed error, %	Flutter frequency error, %
Symmetric modes	*pk* Chebyshev [4, 2]	0.01	0.00	0.01	0.00
	pk Chebyshev [5, 3]	0.01	0.00	0.01	0.00
	pk Chebyshev [6, 4]	0.00	0.00	0.00	0.00
Antisymmetric modes	*pk* Chebyshev [4, 2]	0.08	0.02	0.03	0.02
	pk Chebyshev [5, 3]	0.01	0.00	0.01	0.00
	pk Chebyshev [6, 4]	0.01	0	0.04	0.00

modes at Mach number $M = 1.6$ and for the F/A-18 aircraft antisymmetric modes at Mach numbers $M = 1.4$ and 1.6 show the same trends. The 14 reduced frequencies $k = 0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.588, 0.625, 0.67, 0.71, 0.77, 0.83, 0.91$, and 1 are considered in this study and the indices of k's range are denoted on the horizontal axes of this figure.

In addition to Fig. 1, the numerical approximation error values are further represented in Table 1 for the same two Mach numbers $M = 1.4$ and 1.6, and for three approximation orders: [4, 2], [5, 3], and [6, 4]. The results for the approximation order = [6, 4] are thus represented in two different formats (Fig. 1 and Table 1).

As seen in Table 1, the total normalized approximation error is much smaller using the Chebyshev polynomial method with respect to the overall approximation error given by the Padé method. We notice that the [6, 4] polynomial approximation order already gives a very small approximation error, and for this reason, the Chebyshev method is applied for a maximum of [6, 4] polynomial approximation order (and not for anything higher than this order).

An almost constant and very small approximation error value is found for the reduced frequency interval by use of the Chebyshev approximation method for the [6, 4] model order. Using the Padé method, the approximation error is found to be small (if calculated separately for each reduced frequency) mainly in the middle of the reduced frequency interval, and it increases at each end of this interval. As a result, the total normalized approximation error provided by Padé is much higher than the error obtained using Chebyshev theories, as seen in Table 1.

Another type of result is the one presented for the flutter *pk* analysis. Table 2 shows the speeds and frequencies where an aircraft becomes unstable due to flutter, using the *pk* flutter standard and the *pk* Chebyshev method (for three approximation orders). We have to mention here the fact that the *pk* Chebyshev method is actually the *pk* flutter method in which the aerodynamic forces are approximated in the Laplace domain by use of Chebyshev method. These studies are presented for the symmetric and antisymmetric modes of an F/A-18 aircraft. Three approximation orders ([4, 2], [5, 3], and [6, 4]) and two Mach numbers ($M = 1.4$ and 1.6) are taken into account in the Chebyshev method. These approximation orders and Mach number values are the same as those considered in Table 1.

Flutter analysis results expressed in terms of speeds and frequencies calculated by our method were found to be very close to the initial flutter results obtained by the *pk* flutter standard method. The comparison between these results is shown in Table 3 by use of the following error representation:

$$\text{Error } \% = \frac{|\text{Value}\,pk\text{Standard} - \text{Value}\,pk\text{Chebyshev}|}{\text{Value}\,pk\text{Standard}} \times 100\% \tag{18}$$

where "Value" is the flutter speed or the flutter frequency represented in the first and second columns, for $M = 1.4$ and 1.6, respectively.

In Table 1, it can be seen that the approximation's normalized total error calculated by the Chebyshev method quickly stabilizes around 1–2% (for the approximation order [6, 4]).

Therefore, it has been found that an approximation order of [6, 4] (see Table 3) would be sufficient in the Chebyshev method applied to the F/A-18 aircraft's data to calculate flutter velocities and frequencies. For higher-order approximations, the results remain the same as those obtained for the [6, 4] approximation order.

The reader should not overlook the fact that the accuracy normally obtained using an increased approximation order could be overweighted by computational truncation errors. Each investigator should choose the approximation order best suited for the data to be computed. Our method is an effort to find the best compromise between computational costs and accuracy.

VI. Conclusions

By using the Chebyshev method on the F/A-18 aircraft, regardless of which approximation order was used, the error for flutter speeds and frequencies was small: less than 0.08%. The computation time for the Chebyshev method is up to four times shorter than for the Padé method.

Therefore, we concluded that the Chebyshev method gives better overall results than the Padé method, and based on our previous application on the ATM in STARS, we concluded that it remains an efficient method independent of the aircraft type.

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