JOURNAL OF AIRCRAFT
Vol. 44, No. 6, November–December 2007

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Ground Dynamics Model Validation by Use of Landing Flight Test

M. Nadeau Beaulieu,* R. M. Botez,† and A. Hiliuta‡
École de Technologie Supérieure, Montréal, Québec H3C 1K3, Canada

DOI: 10.2514/1.26625

Nomenclature

C_L	=	roll damping
C_M	=	pitch damping
C_N	=	yaw damping
C_z	=	vertical damping
F_K	=	kinetic friction coefficient
F_S	=	static friction coefficient
F_x	=	longitudinal force
F_y	=	lateral force
F_z	=	vertical force or normal force between the helicopter and the ground
K_i	=	constant, where i is an index = 1, 2, 3
K_L	=	roll stiffness
K_M	=	pitch stiffness
K_z	=	vertical stiffness
L	=	rolling moment
M	=	pitching moment or mass
N	=	yawing moment
t	=	time
V_{tan}	=	resultant velocity tangential to the ground
x, y, z	=	axes coordinates
$\dot{z}$	=	vertical velocity
$\ddot{z}$	=	vertical acceleration
β	=	exponential decay coefficient
Δt	=	time increment
Δz	=	vertical skids deflection
θ	=	pitch angle
$\dot{\theta}$	=	pitch angle rate of change
λ	=	angle between the longitudinal and the resultant velocity components
ν	=	friction coefficient
ϕ	=	roll angle
$\dot{\phi}$	=	roll angle rate of change
ψ	=	yaw angle
$\dot{\psi}$	=	yaw angle rate of change

Subscripts

CG	=	center of gravity
eq	=	at equilibrium (at the end of the landing record)
friction	=	from friction
gravity	=	from gravity
GD	=	from ground dynamics
lever arm	=	lever arm between the touchdown point and the center of gravity
OEI	=	one-engine inoperative
oscillations	=	from the "oscillations" equations
pivot	=	from the "pivot" equations
thrust	=	from thrust

Presented as Paper 6811 at the AIAA Guidance, Navigation, and Control Conference and Exhibit, Keystone Resort and Conference Center, Keystone, Colorado, 21–24 August 2004; received 18 July 2006; revision received 4 July 2007; accepted for publication 5 July 2007.

*Ph.D. Student, Laboratory of Research in Active Control, Avionics and Aeroservoelasticity, Automated Production Engineering Department 1100, rue Notre-Dame Ouest.

†Professor, Laboratory of Research in Active Control, Avionics and Aeroservoelasticity, Automated Production Engineering Department 1100, rue Notre-Dame Ouest.

‡Postdoctoral Fellow, Laboratory of Research in Active Control, Avionics and Aeroservoelasticity, Automated Production Engineering Department 1100, rue Notre-Dame Ouest.

I. Introduction

THIS Note presents a ground dynamics model which reproduces the helicopter motion felt by the pilot during a landing. The model validation was realized by comparison between the helicopter's orientation angles and ground speed time histories with landing data time histories. The model performance was found to be within the tolerances set by the Federal Aviation Administration (FAA) for one-engine inoperative (OEI) and autorotation landings. Our mathematical model improved understanding of the helicopter's behavior during touchdown. The theory and principles developed in this work could be used to validate other helicopter landings.

Because there is limited available literature on helicopter ground dynamics, we here refer to rigid body collision and contact theories for mathematical formulations of an impact between an object and the ground. Two classes of methods may be used: the *impulse method* and the *penalty method* which are described next.

The impulse method [1] uses instantaneous impulses to model the impact between rigid body and the ground. Following touchdown, when the rigid body stays on the ground, the forces exerted on it can be represented by analytically computed constraint forces or by multiple impulses that insure that no part of the body penetrates the ground. This class of method needs to evaluate the exact collision time between the body and the ground.

Secondly, the penalty method consists of modeling the ground as a spring that pushes the body upwards as it penetrates into the ground. Here the body penetrates slightly into the ground, but there is no need to evaluate the exact touchdown time. This method is used because is the simplest one to implement.

In fact, the penalty method [2] was used by Johnson to model the ground dynamics of an unmanned helicopter because the small ground penetration of the helicopter touchdown point could be interpreted as skid deformation. Johnson did not specify how the stiffness and damping of each spring were selected. This unmanned helicopter model was not validated by use of landing test data.

II. Methodology

A. Global Model Simulation Structure

The implementation of the ground dynamics model in the global simulation is shown in Fig. 1. As shown in this figure, the global model uses inputs related to the helicopter dynamic conditions (velocities, Euler angles etc.) for forces and moments calculations. These forces and moments are then inputs to the 6 degree-of-freedom helicopter equations of motion to obtain the rates of change in its dynamic conditions. These rates of change are used to update the dynamic conditions for the next time step ($t + \Delta t$).

The global model includes the calculation on the ground of the forces and moments from the thrust and weight and the ground. This model is activated with a logical touchdown flag when a touchdown is detected. This detection is done by calculating the vertical distance between the ground and the lowest point on the helicopter skids in the modified Earth-axis coordinate system. This coordinates system is defined by successive rotations of the body axes coordinates system in roll and pitch to bring the x and y axes parallel to the Earth and insure that the z axis points toward the ground.

B. Simplified Thrust and Weight Model

In this Note, the following assumptions are made for the aerodynamic, thrust, and gravity forces and moments at touchdown:

1) The aerodynamic moments are negligible with respect to the moments from the ground contact and helicopter oscillations. Therefore, the moments from the stability derivatives are neglected and the aerodynamic moments from the control derivatives are progressively reduced as the main rotor thrust decreases and the normal forces between the ground and the helicopter increase.

2) The aerodynamic drag forces are negligible in comparison with the ground friction forces.

3) The resultant gravity and thrust forces in the x and y directions are zero.

Following these assumptions, the only significant forces and moments after touchdown that remain are the vertical forces from thrust, the weight forces, and the ground interaction described next.

C. Normal and Friction Forces from Ground Dynamics

When the helicopter touches down, the ground reaction is represented by a spring that pushes the helicopter up. We can therefore represent the helicopter's vertical motion by the equation of a spring-mass system:

$$\sum F_{z,\text{resultant}} = M\ddot{z} = F_{z,\text{Gravity}} + F_{z,\text{Thrust}} - C_z\dot{z} - K_z\Delta z \quad (1)$$

where $\sum F_{z,\text{resultant}}$ is the total force acting on the helicopter in the z direction and the ground dynamics part of Eq. (1) reduces to the stiffness and damping terms which can be interpreted as skid deformations:

$$F_{z,\text{GD}} = -C_z\dot{z} - K_z\Delta z \quad (2)$$

In Eq. (2), K_z and C_z are functions of the skids' deflection and the helicopter pitch angle on the ground. These dependencies were found by a detailed inspection of the landing data and are thoroughly explained in a more detailed version of this Note [3]. The magnitudes of the friction forces are expressed in the following equation:

$$F_{\text{friction}} = -\nu F_{z,\text{GD}} \quad (3)$$

The resulting forces acting in the x and y directions from friction can be expressed as follows:

$$F_{x,\text{GD}_{\text{friction}}} = -F_{\text{friction}} \cos\lambda = -\nu F_{z,\text{GD}} \cos\lambda \quad (4a)$$

$$F_{y,\text{GD}_{\text{friction}}} = -F_{\text{friction}} \sin\lambda = -F_{z,\text{GD}} \sin\lambda \quad (4b)$$

where λ is the angle between the longitudinal and resultant components of the helicopter's velocity. The friction coefficient ν increases with the decreasing helicopter velocity tangential to the ground $V_{\tan}$ and is expressed [4] by Eq. (5):

$$\nu = F_K + (F_S - F_K)e^{-\beta V_{\tan}} \quad (5)$$

From the landing data it is found that the static friction coefficient F_S is equal to 0.4, which corresponds to the values range for concrete to steel materials (0.30–0.70) found in the literature [5]. The kinetic friction coefficient F_K and the exponential decay coefficient β are dependent upon the magnitudes of the helicopter's oscillations in roll and pitch at the touchdown time [3].

D. Rolling and Pitching Moments from Ground Dynamics After Touchdown

When the helicopter touches down, the contact and friction forces introduce rolling and pitching moments that rotate the helicopter toward the ground (see Fig. 2). In this Note, these moments are called *pivot* moments. Following the initial rotation, the helicopter keeps oscillating as shown in Fig. 3, and these moments are referred to as *oscillation* moments.

1. *Moments During Initial Pivot Phase*

The pivot rolling moments L_{pivot} can be represented by the following equation:

$$L_{\text{pivot}} = F_{z,\text{GD}}\, y_{\text{lever arm}} - C_{L,\text{pivot}}\dot{\phi} \quad (6)$$

where $F_{z,\text{ground}}\, y_{\text{lever arm}}$ is the vertical force in the z direction times the lateral distance between the touchdown point and the center of gravity of the helicopter, and $C_{L,\text{pivot}}\dot{\phi}$ is a damping term where $C_{L,\text{pivot}}$ is a damping coefficient. This damping coefficient $C_{L,\text{pivot}}$ is considered as a quadratic function of the roll rate of the helicopter on the ground [3].

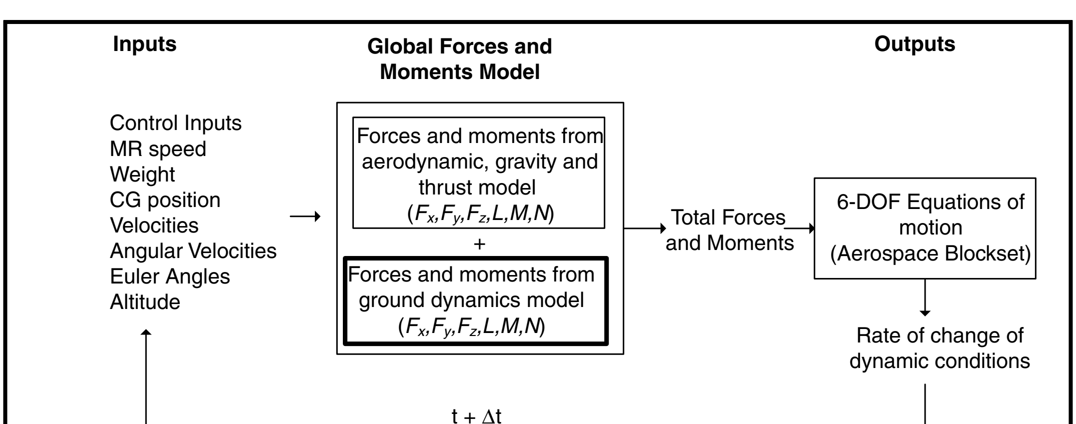

Fig. 1 Implementation of the ground dynamics model in the global simulation.

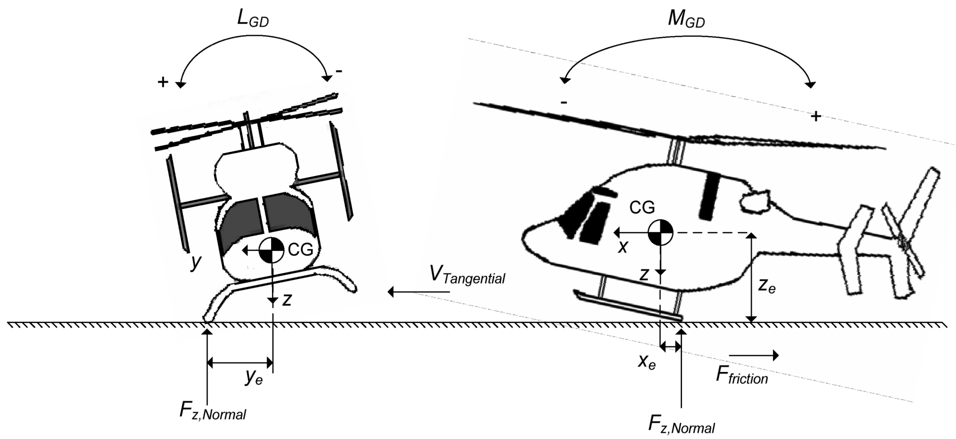

Fig. 2 Rolling and pitching pivot moments during the initial rotation.

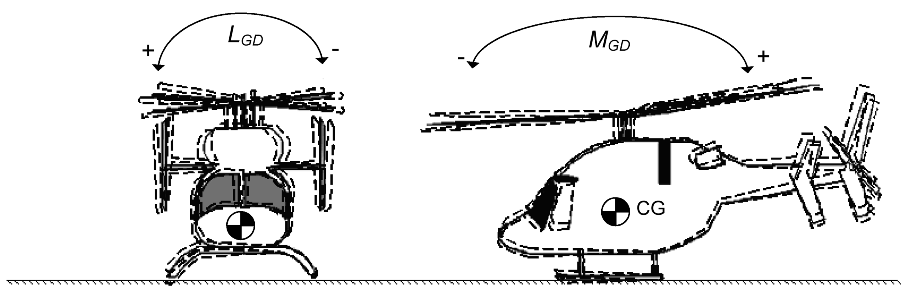

Fig. 3 Rolling and pitching oscillation moments following the initial rotation.

The pivot pitching moment M_{pivot} can be determined by the use of Eq. (7):

$$M_{\text{pivot}} = -K_1(1 + K_{\text{pitch rate}})F_{z,\text{GD}}x_{\text{lever arm}} + K_2 F_{x,\text{GD}} z_e - C_{M,\text{pivot}}\dot{\theta} \tag{7}$$

where the term $-F_{z,\text{ground}}x_{\text{lever arm}}$ represents the product between the force in the z direction and the longitudinal distance $x_{\text{lever}_{\text{arm}}}$ between the touchdown point and the helicopter center of gravity. The term $F_{x,\text{GD}}z_e$ in Eq. (7) represents the product between the friction force in the x direction and the vertical distance between the touchdown point and the center of gravity. The third term $-C_{M,\text{pivot}}\dot{\theta}$ is a damping term where $C_{M,\text{pivot}}$ is the damping coefficient. The K_1 and K_2 terms are the optimized coefficients which insure the matching of the model with the landing test data. The correction factor $K_{\text{pitch rate}}$ is a correction factor for a high positive pitch rate at touchdown and is explained in detail in [3].

2. Moments During the Oscillation Phase

Following the initial rotation, the helicopter oscillates around its equilibrium pitch and roll angles θ_{eq} and ϕ_{eq} as shown in Fig. 3. The moments from these oscillations are found by use of the following equations:

$$L_{\text{oscillation}} = -K_L(\phi - \phi_{\text{eq}}) - C_L(\dot{\phi} - \dot{\phi}_{\text{eq}}) \tag{8}$$

$$M_{\text{oscillation}} = -K_M(\theta - \theta_{\text{eq}}) - C_M(\dot{\theta} - \dot{\theta}_{\text{eq}}) \tag{9}$$

We emphasize here that in these equations, no specific stiffness or damping components are represented, but the overall rolling and pitching motion resistance is represented in these equations when the helicopter is on the ground. Expressions for the stiffness and damping coefficients in the above equations are derived to match the helicopter's landing data [3]. These coefficients are a nonlinear function of some of the helicopter dynamic parameters such as its forward speed, pitch angle, and pitch rate, as they are empirically found from a thorough inspection of the helicopter behavior after touchdown from a different landing case.

For example, the values of stiffness and damping required to match a landing case are different for low-speed and high-speed landings. The appropriate approach in this case is therefore to express the stiffness and damping as a function of the helicopter's velocity in order to match both landing records. A similar empirical approach was used to find expressions for the other coefficients. After touchdown, because the roll center of the helicopter is not necessarily at the same position as the center of gravity, when the helicopter oscillates in roll, the pilot feels a sideward acceleration. This acceleration is reproduced in the model by an oscillating sideward force in phase with the helicopter roll angle oscillation and is represented by the following equation:

$$F_{y,\text{GD,oscillation}} = K_3 L_{\text{oscillation}} \tag{10}$$

This force is added to the friction force in the y direction given by Eq. (4b). There was no such relationship observed with the force in the x direction.

E. Yawing Moment from Yaw Damping Term $N_{\text{GD damping}}$

The yawing moment N from the ground is given by a damping term due to friction and a coupling term from the rolling oscillations. The yawing moment can therefore be expressed by Eq. (11):

$$N_{\text{GD damping}} = -C_N\dot{\psi} + K_3 L_{\text{oscillation}} \tag{11}$$

where the first term is a damping term and the second term is a coupling term with the helicopter's rolling motion. This coupling term is added because, in the flight data, the helicopter's yaw oscillations are in phase with its roll oscillations. Contrary to the roll and pitch equations, there is no heading equilibrium position and therefore, there is no stiffness term. The general expression for the yaw damping coefficient varies with the normal force between the helicopter and the ground and the helicopter pitch angle [3].

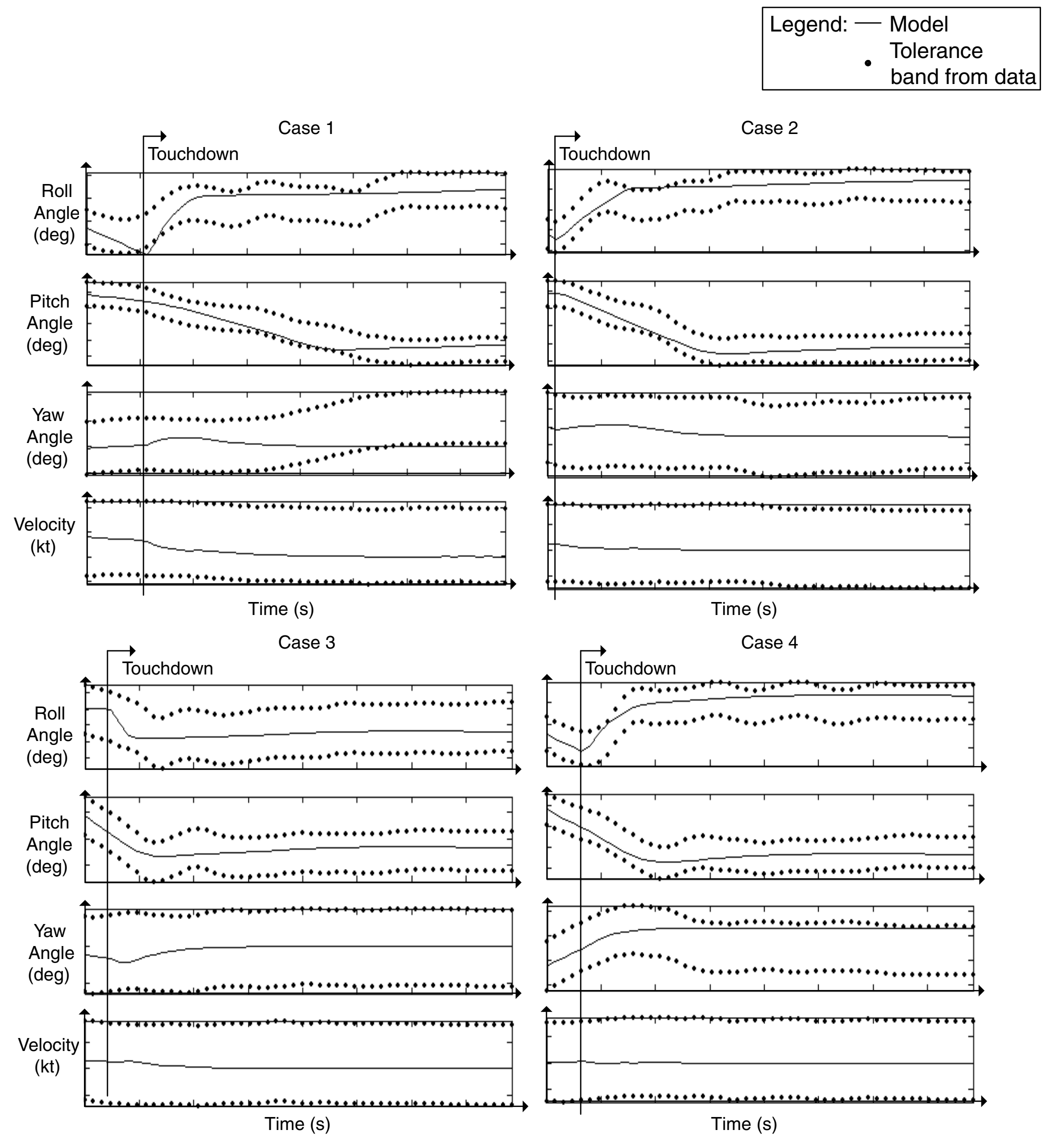

Fig. 4 Roll, pitch, yaw angles and velocity versus time for four OEI landing cases.

III. Results

The ground dynamics model was validated for 14 landing cases, but results obtained with this model were presented for eight typical landing cases. The six other landing cases are similar and, for this reason, are not shown. In Figs. 4 and 5, the model outputs are represented by solid lines and the dotted lines represent the FAA tolerance bands, which are as follows [6]:

1) one-engine inoperative (four cases presented in Fig. 4):

$$\phi = \pm 1.5 \text{ deg}; \qquad \theta = \pm 1.5 \text{ deg}$$
$$\psi = \pm 2 \text{ deg}; \qquad V_{\tan} = \pm 3 \text{ kt}$$

2) autorotation (four cases presented in Fig. 5):

$$\phi = \pm 2 \text{ deg}; \qquad \theta = \pm 2 \text{ deg}$$
$$\psi = \pm 5 \text{ deg}; \qquad V_{\tan} = \text{no tolerance}$$

A. Results for One-Engine Inoperative Landings

Figure 4 shows the time histories of the roll angles ϕ, pitch angles θ, yaw angles ψ, and velocity $V_{\tan}$ for four4 OEI landing cases. Please note that no numbers are shown on the x and y axes for confidentiality reasons. From Fig. 4, it is clear that the ground dynamics model outputs are within the FAA tolerance bands. Notice that the scale of the y axis was adjusted on each figure to enhance readability. In these cases, the pivot rolling and pitching moments return the helicopter slowly to its equilibrium position. The oscillation moments equations are then used to stop the helicopter's motion. It can also be observed that the tangential velocity $V_{\tan}$ decreases slowly to zero due to the friction, and the rate of change of the yaw angle slowly decays due to the yaw damping.

B. Results for Autorotation Landings

Results for four out of the seven autorotation cases that were used to validate the model are shown in Fig. 5. To improve the readability of the results, the time length of the velocity plot is 3 times longer than the time scale on the angles plot. The y-axis scale was adjusted on each figure to enhance readability. In these autorotation cases, the results are found within the FAA tolerance bands. The results for the other cases are not displayed, but they are very similar to the results displayed in Fig. 5. It can be observed that the model is mostly driven by the helicopter oscillation equations because the initial rotation is done quickly.

IV. Conclusions

A new formulation for the helicopter ground dynamics was developed based on one-engine inoperative and autorotation flight data. A spring with stiffness and damping was used to calculate the normal forces on the helicopter at touchdown and a friction equation was used to model the helicopter's speed decay. The rolling and

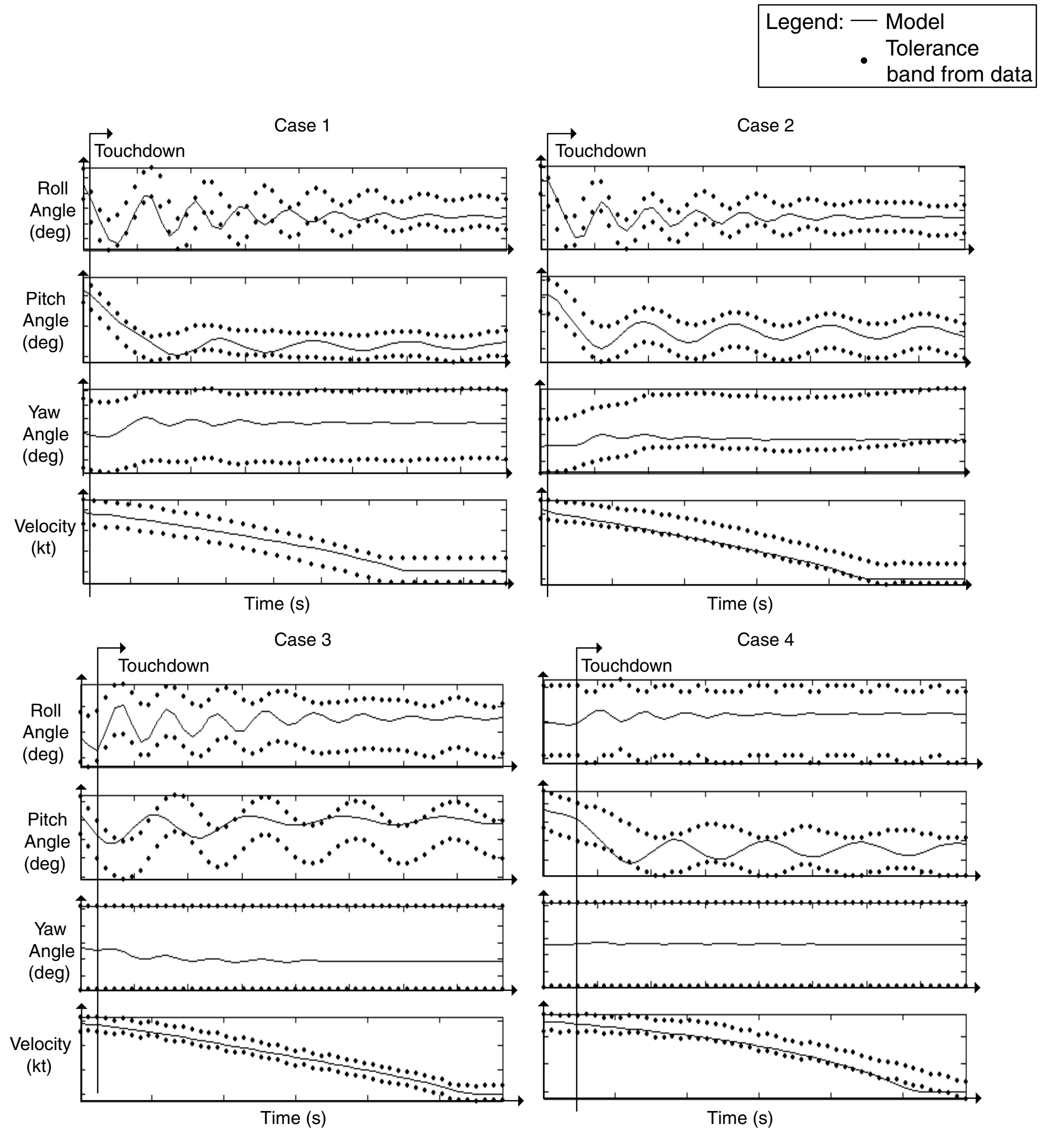

Fig. 5 Results for four autorotation landing cases.

pitching moments were calculated during the initial rotation from the offset of the ground dynamics forces with respect to the helicopter center of gravity. After the initial rotation, these moments came from the torsional stiffness and damping of the helicopter after touchdown. The yawing moment at touchdown was computed with a damping term that varied with the normal force between the ground and the helicopter and a coupling term that related it to the oscillation rolling moments. The model output shows good agreement with the landing data for both OEI landing cases (see Fig. 4) and autorotation landing cases (see Fig. 5).

References

[1] Baraff, D., "Analytical Method for Dynamic Simulation of Non-Penetrating Rigid Bodies," *Computer Graphics*, Vol. 23, No. 3, 1989, pp. 223–232. doi:10.1145/74334.74356

[2] Johnson, E. N., and DiBietto, P. A., "Modeling and Simulation for Small Autonomous Helicopter Development," AIAA Paper 97-3511, Aug. 1997.

[3] Nadeau Beaulieu M., Botez R. M., and Hiliuta A., "Ground Dynamics Model Validation by Use of Landing Flight Test Data," AIAA Paper 06-6811, Aug. 2006.

[4] Sareen, A. K., Smith, M. R., and Howard, J. V., "Helicopter Skid Gear Dynamic Drop Analysis and Test Correlation," *American Helicopter Society 54th Annual Forum*, American Helicopter Society, Alexandria, VA, 20–22 May 1998, pp. 1267–1274.

[5] Beer, F. P., and Johnston, R., *Vector Mechanics for Engineers, Static and Dynamics*, 7th ed., McGraw–Hill, New York, 2003.

[6] "Helicopter Simulator Qualification," Federal Aviation Administration, Advisory Circular AC 120-63, Nov. 1994.