Generation of Signatures by Deformations

Claudio de Oliveira[1], Celso A A Kaestner[1,2], Flávio Bortolozzi[1] e Robert Sabourin[3]
Pontifícia Universidade Católica do Paraná - PUC-Pr.[1]
R. Imaculada Conceição,1155 - 80215-901 – Curitiba -Pr.
Centro Federal de Educação Tecnológica do Paraná - CEFET-Pr[2]
Av. 7 de Setembro, 3165 - 80230-901 - Curitiba, Pr. Brasil
École de Technologie Supérieure[3] - 1100, rue Notre-Dame Ouest
Montréal (Québec) H3C 1K3, Canadá
claudio@rla01.pucpr.br[1], *fborto@rla01.pucpr.br*[1] *sabourin@gpa.etsmtl.ca*[3]

ABSTRACT: The techniques of automatic classification of hand-written signatures have been studied and some of them are based on the application of neuronal nets or statistical methods. Nevertheless, the great number of samples required by these methods turns many of its practical applications unfeasible. This article describes a technique for automatic generation of signatures originated from the deformation of a reduced number of genuine samples. The technique used here is based on convolution between deforming polynomials representing the deformations and the signals representing the horizontal and vertical moves of the pen, required for the reproduction of the original samples. The result of the convolution produces the deformation of those signals and, consequently, the deformation of the tracing obtained from them.

1. INTRODUCTION

In signatures off-line acquisition every dynamic information relevant to the writing process is lost, since the acquisition occurs after the writing is done.

We use in our work the off-line signature acquisition and consequently the tracing sequentialization is lost. However, this characteristic is of major importance for the development of the proposed signature deformation technique. The partial reconstruction of the normal writing tracing sequentialization starting from off-line samples is described by [DOE93]. Although the algorithms for re-connection of the normal writing tracing produce good results, they are not perfect. Thus, in spite of the feasibility of the adaptation of those algorithms to the signature writing, deeper studies for the development of automatic signature deformation tools will be required. These tools can be found at [PLAM89], [PLAM90], [RAND90] and [SABO90].

The steps required for the obtaining the sequence of points defining the signature tracing are thinning, shrinking, determination of characteristic points, determination of trace segment, reconnection and trace sequencialization.

A sample of input signature can be seen on

Fig. 1.

Considering that the signature samples obtained by the off-line acquisition process have been previously filtered (that is, without noise) and threshold (binary), the stage of samples pre-processing consists of the re-construction of the sequentialization of the tracing.

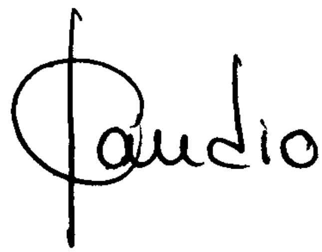

Fig. 1: Sample of input signature

2. IMPORTANT CONCEPTS

2.1 - Thinning

To obtain a sequence of points which will allow the following of the trace it is necessary that each point, except extreme and trace-crossing points, have a predecessor and a successor. This restriction does not apply to the component points of the image obtained soon after the signature acquisition. Thus, it is necessary to select the component points of the signature skeleton. The thinning algorithm produces a trace representation complying with that restriction. Each trace segment of the thinned image is exactly 1 pixel thick. The choice of the thinning algorithm is of major importance, since the performance of the following algorithm depends directly on the quality of the resulting skeleton. **Fig. 2** shows the thinned image corresponding to the sample image and the improper points originated by most thinning processes.

2.2 - Shrinking

The improper points generated by the thinning process are normally 3 pixels long. The controlled application of three interactions of the thinning algorithm erases three pixels from each segment ending, including those that form a normal tracing. The loss of three or four pixels of normal segments is only relevant if the segment is too small. In these cases, the segment may be completely erased.

The disappearance of small segments does not produce significant losses in the signatures modelling, normally composed by lots of points. However, these segments can define characteristic points determining the beginning or ending of real segments and, in these cases, should not be eliminated. **Fig. 3** shows the thinned image without the improper points that result from the thinning process.

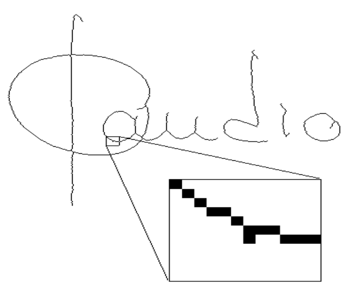

Fig. 2 Thinned image Fig. 1

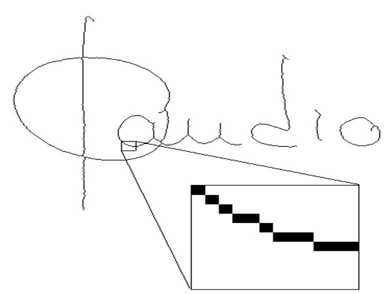

Fig. 3 Result thinning process.

2.3 - Characteristic Points

After the thinning, points in the image that delimit the trace segments can be observed. These characteristic points are either extreme points or points where trace crossing of two or more traces occur, as shown in **Fig. 4**.

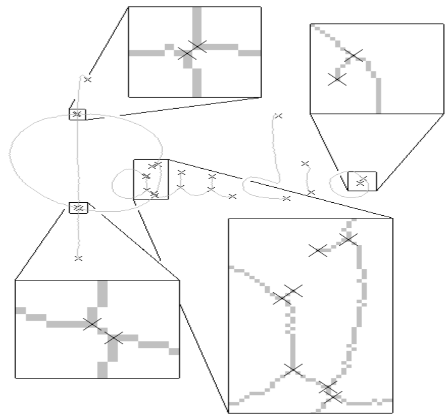

Fig. 4. Characteristics points.

2.4 - Trace Segments

A trace segment is a nonempty sequence of points between two characteristic points either equal or different. The sequence of points which define a trace segment do not necessarily represent the sequence of the tracing. At this stage of the pre-processing, only the set of points defining the segment and the neighbourhood of each of these points are determined.

The algorithm for extraction of trace segments considers that there are not two or more pixels associated to the same characteristic points, as shown in **Fig. 5**(a). The result of the removal of the undesirable pixel is shown in **Fig. 5**(b).

The elimination of these and other similar pixels does not allow the characteristic points associated to the trace crossing to have more than one pixel. As a consequence, in the crossing of two or more traces, only one pixel is defined.

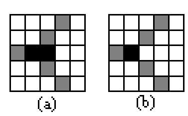

Fig. 5 . The result of the removal of the undesirable pixel

Considering the restriction described on the previous paragraph, the algorithm for extraction of segments analyses the neighbourhood of each of the characteristic points. For each non analysed neighbour of each characteristic point, one segment is initiated. The subsequent point of the sequence is determined by analysis of the neighbours of the last point included in the sequence. In this case the last point and its two closest neighbours are not considered, in compliance with the trace direction. This procedure is repeated until there is not neighbour of the current point. In the event of reaching an ending or a crossing or in case two or more neighbours occur, a segment is identified and finalised.

For each determined segment two neighbours of characteristic points are analysed: one at the beginning and other at the end of the determination. The algorithm memorises the neighbours of the characteristic points reached at the end of the segment determination, not re-analysing them in the determination of the beginning of new segments. This prevents the duplicity of segments to occur. **Fig. 6** shows the decomposition of the sample signature in trace segments.

2.5 - Reconnection and sequentialization

In the experiments to be described, both the reconnection and the sequentialization of the tracing are manually done. It is possible to select the segments that compose a trace and to assign a course to the trace, starting from the trace segments and the original image. Both the selection of segments and the assignment of course are heuristic, but produce good results when a pre-selection of the signature samples is made.

Once the segments that compose a trace are selected, they may be put together, if necessary, in the endings corresponding to the points that belong to equal crossings, thus forming a unique sequence of points corresponding to the whole trace. After that, this sequence, unique for each trace, may have to be inverted to reflect the tracing sequentialization.

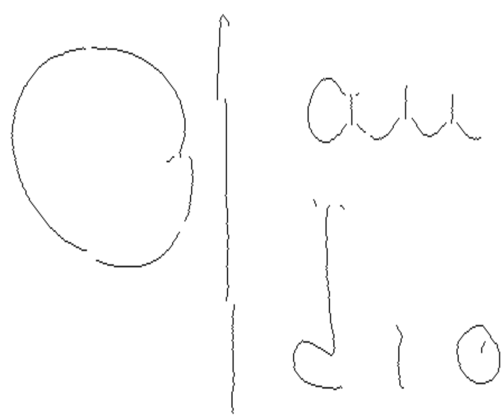

Fig. 6 . The decomposition of the sample signature in trace segments

It may occur that the same segment belongs to two or more traces. This is due to the superposition of traces or to the appearance of the «false» segments, previously discussed. This does not cause problems in the manual reconnection and sequentialization, since they require human decisions. **Fig. 7** shows the result of the segments composition.

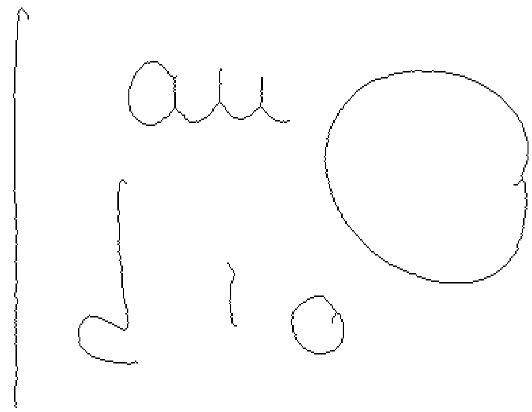

Fig. 7 . The result of the segments composition.

2.6 - Traces Mass Centres

The trace mass centre (x_o, y_o) is defined by the equation:

$$(x_0, y_0) = \left(\frac{1}{n} \sum_{i=1}^{n} x_i , \frac{1}{n} \sum_{i=1}^{n} y_i \right)$$

Fig. 8 shows the mass centres of the signature shown on **Fig. 1**.

After finding the mass centre (x_o, y_o) of a trace, all of its points must be translated to $(x'_i, y'_i) = (x_i - x_o, y_i - y_o)$, which permits the individual processing of traces, avoiding the positional dependency between traces.

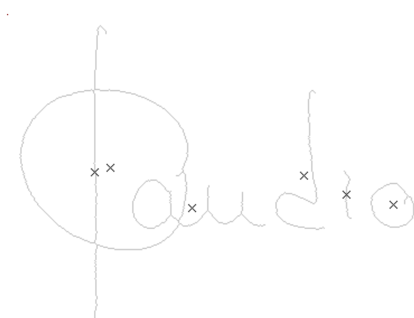

Fig. 8 . The mass centres of the signature shown on
Fig. 1

2.7 - Reconstruction of Tracing Movements

The time unit used as a system of co-ordinates in our work, has a value equal to the value of the distance, either horizontal or vertical, between to consecutive points of the trace, in accordance with the type of movement. This time system of co-ordinates represents the forms of the two movements which will have a duration, in time units, equal to the number of points in the trace.

After definition of the time system of co-ordinates, from the first point of the sequence representing the trace, its horizontal or vertical co-ordinates are ordinately projected according to the sequentialization assigned to the tracing, in an vertical or horizontal axis, in accordance with the movement being obtained.

Perpendicular to the projection axes, the axes representing time keep the information of sequentialization, associating to each point projection, the position in which it occurs in the sequence. It is noticed that the duration of the tracing is equal, in time units, to the number of points of the trace, and, thus, has the only purpose of maintaining the co-ordinates of the points which constitute the trace duly sorted, in compliance with the order of occurrence of points during the writing process. Figure 11 illustrates the movements obtained from one trace of the sample signature. It is noticed that the tracing starts at point *a* and describes the sequence < *a*, *b*, *j*, *c*, *k*, *d*, *l*, *e*, *m*, *f*, *n*, *g*, *o*, *h*, *p*, *i* >.

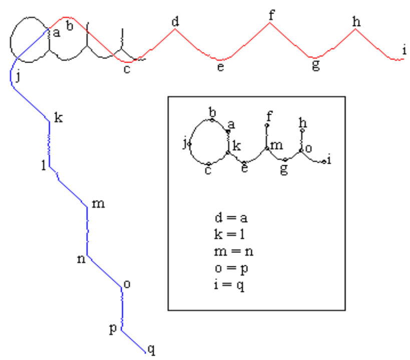

Fig. 9 . The movements obtained from one trace of the sample signature.

Due to the choice of the time system of co-ordinates, the movements can be easily represented. As the time simply reflects the position of points within the trace, the movements can be represented as sequences of co-ordinates with the same order of the sequence of points that define the trace.

If the sequence $\langle (x'_0, y'_0), (x'_1, y'_1), \ldots, (x'_{n-1}, y'_{n-1}) \rangle$ represents a trace, then the sequences $\langle x'_0, x'_1, \ldots, x'_{n-1} \rangle$ and $\langle y'_0, y'_1, \ldots, y'_{n-1} \rangle$ represent, respectively, the forms of horizontal and vertical movements associated to it.

The sequences of co-ordinates are interpreted as polynomial of order n-1, where n is the number of points of the trace. For example, the polynomial obtained from $\langle x'_0, x'_1, \ldots, x'_{n-1} \rangle$ and $\langle y'_0, y'_1, \ldots, y'_{n-1} \rangle$ are $x'_0 t^0 + x'_1 t^1 + \ldots + x'_{n-1} t^{n-1}$ and $y'_0 t^0 + y'_1 t^1 + \ldots + y'_{n-1} t^{n-1}$, respectively.

3 - Deformations of signatures

The convolution describes the action of an observation instrument when it produces a weighted average of a range of some physical quantity. As the form of the function of pondering does not change significantly, the observed quantity is a value of the convolution of the distribution of the desired quantity with the pondering function, instead of the value itself. In this work, the movements representing the traces of the signatures will be considered as physical quantity and after this, it will be described how simple pondering functions modify those traces.

3.1 – Convolution

The convolution of an function that can be integrated *f(x)* and of a sample function $g(x)$ called instrument, is given by :

$$h(x) = \int_{-\infty}^{+\infty} f(u)g(x-u)du$$

It is important to notice that the convolution is a function of x. However, to determine $h(x)$ in the point x_1, it is needed to know $f(x)$ for all x.

The relationship between the general characteristics of *h(x)* and *f(x)* is illustrated in **Fig. 10**, where it can be observed that *h(x)* is smoother than *f(x)*. A proper choice of *g(x)* can modify, either much or little, the characteristics of *f(x)*.

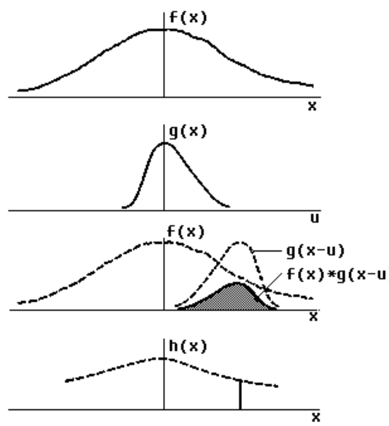

Fig. 10 . The relationship between the general characteristics of *h(x)* and *f(x)*.

3.2 - Serial Product

The polynomial representing the movements obtained from the pre-processing of the signature images are discrete. Thus, *f(x)* cannot be evaluated for every x. However, the approximate convolution can be obtained in the form of a serial product. Considering the polynomial

$$A = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$
$$B = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \ldots$$

Thus, the serial product $A*B$ is given by:

$$A * B = \sum_{i=0}^{n} c_i x^i \text{ , where } c_i = \sum_{j=0}^{i} a_j b_{i-j} \text{ .}$$

The product has an important connection with the convolution. Suppose that two functions f and g are given, and it is necessary to numerically calculate its convolution. Then if f and g are the sequences of values $< f_0, f_1, f_2, f_3, \ldots, f_m >$ and $< g_0, g_1, g_2,$

g_3, ... , g_n > in intervals of same short duration w, then the integral of approximate convolution $f*g$ can be obtained by adding products of corresponding values of f and g.

3.3 - Uniform deformations through degree instruments

A uniform deformation of signature is that in which the deformation is constant for all the points of the signature. In the discussions that follows, the following conventions are accepted:

- $f_V(t)$ and $f_H(t)$ denote, respectively, the polynomials representing the vertical and horizontal sequences of co-ordinates, in time t, described by the pen during the writing of a trace T;
- $g_V(t)$ and $g_H(t)$ are the instruments (or sampling functions) to be used for the deformation of the forms of vertical and horizontal movements, respectively. Such instruments will be generically called deforming polynomials;
- $h_V(t)$ and $h_H(t)$, with $h_V(t) = f_V(t) * g_V(t)$ and $h_H(t) = f_H(t) * g_H(t)$, denote, respectively, the polynomials representing the vertical and horizontal co-ordinates, in time t, that the pen must describe to generate the deformed trace T;
- $f(t)$ generically denote $f_V(t)$ or $f_H(t)$, and $g(t)$, $g_V(t)$ or $g_H(t)$;
- f_{t_1} and g_{t_1} are simplifications of $f_V(t_1)$, or $f_H(t_1)$, and $g_V(t_1)$, or $g_H(t_1)$, depending on the context; and
- the width w of the intervals between the acquisition of two consecutive co-ordinates, horizontal or vertical, is equal to the time unit.

3.3.1 - Scale

Considering the deforming polynomial of order 0, $g(t)=g_0$, where g_0 is constant. The serial product between $f(t)$ and $g(t)$ is the sequence $h(t)=<f_0*g_0, f_1*g_0, f_2*g_0, ..., f_m*g_0>$ which can also be written as $h(t)=<f_0, f_1, f_2, ..., f_m>*g_0$ or $h(t)=g_0*f(t)$. It is clearly noticed that, in these conditions, the reduced equation for $h(t)$ produces a uniform modification of the amplitude of $f(t)$. Applying this thinking more specifically to $f_V(t)$ and $f_H(t)$, we reach $h_V(t)=f_V(t)*g_V$ and $h_H(t)=f_H(t)*g_H$, where g_V and g_H are constant polynomials of order 0, deforming respectively and uniformly, the vertical and horizontal amplitudes of the original tracing.

If g_V and g_H have the same value, then the horizontal/vertical proportionality of the tracing is maintained; otherwise, the tracing is flat or stretched in one direction relative to the other, depending on the values of g_V and g_H. If gv or gh or both are negative, the trace is reflected and possibly scaled in one or in both directions.

The equation $(x'', y'') = (x' * g_V, y' * g_H)$ established the relationship between points of the deformed trace (x'', y'') and of the original (x', y'). **Fig. 11** illustrates some examples of uniform scale deformations.

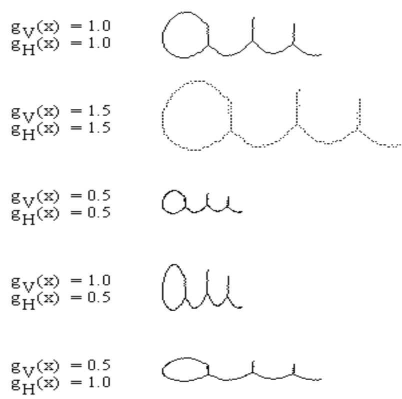

Fig. 11 . Examples of uniform scale deformations.

3.3.2 - Rotation

The rotation of a point $Pt'=(x', y')$, α_o relative to the origin of mass centre, moves this point to $Pt'=(x', y')$, in a way that the distance between $Pt'=(x', y')$ and the origin is equal to the distance between Pt'' and the origin. Before the rotation, the co-ordinates (x', y') define an angle α_1 between the axis x and the straight line passing through the origin and the point Pt'. After the rotation, the co-ordinates (x'', y'') define the angle $\alpha_2=\alpha_1+\alpha$ between the axis x and the straight line passing through the origin and the point Pt''.

In the serial $h_V(t)=f_V(t)*g_V(t)$ and $h_H(t) =f_H(t)*g_H(t)$, the pair $<f_H(t), f_V(t)>$ represents the co-ordinates of the point that occurs in time t before the rotation and, $<h_H(t), h_V(t)>$, the co-ordinates of the same point after the rotation. Thus, the co-ordinates of the rotated points $<h_H(t), h_V(t)>$ should be given by:

$$h_H(t) = f_H(t)*\cos\alpha_H + f_V(t)*\mathrm{sen}\,\alpha_H$$
$$h_V(t) = -f_H(t)*\mathrm{sen}\,\alpha_V + f_V(t)*\cos\alpha_V$$

where the rotation angles α_H and α_V have been differed to allow more flexibility of deformation.

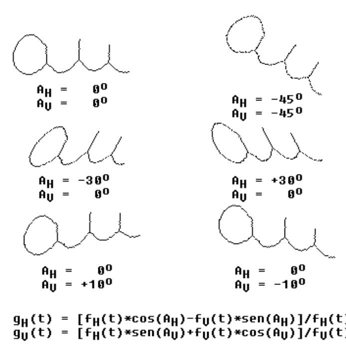

Fig. 12 : Uniform deformations of rotation

Thus the deforming polynomials given by:

$$g_H(t) = \frac{f_H(t) * \cos\alpha_H + f_V(t) * \operatorname{sen}\alpha_H}{f_H(t)}$$

$$g_V(t) = \frac{-f_H(t) * \operatorname{sen}\alpha_V + f_V(t) * \cos\alpha_V}{f_V(t)}$$

The **Fig. 12** illustrates uniform deformations of rotation.

3.3.3 - Scale and rotation

The scale and rotation polynomials can be combined to represent simultaneous deformations of scale and rotation. g_{Ve} and g_{He} being the scale deforming polynomials, and g_{Vr} and g_{Hr} the rotation deforming polynomials, then the polynomials $g_V = g_{Ve} * g_{Vr}$ and $g_H = g_{He} * g_{Hr}$ represent respectively, vertical and horizontal deformations both of scale and rotation. The equations

$$g_H(t) = \frac{f_H(t) * \cos\alpha_H + f_V(t) * \operatorname{sen}\alpha_H}{f_H(t)} * g_H$$

$$g_V(t) = \frac{-f_H(t) * \operatorname{sen}\alpha_V + f_V(t) * \cos\alpha_V}{f_V(t)} * g_V$$

represent the deforming polynomials both of scale (constants g_V and g_H) and rotation (constant angles α_H and α_V).

3.4 - Variable deformations through degree 0 instruments

The deformations are not uniform through all the tracing. Short variations of the parameters used for deformation of scale and/or rotation occur as the tracing evolves. Other more intense variations, practically observable, occur at the beginning and at the ending of the tracing. Thus, in the relationships $h_V(t) = f_V(t) * g_V(t)$ and $h_H(t) = f_H(t) * g_H(t)$, between the deformed trace $h(t)$ and the original $f(t)$ the coefficients of the order 0 polynomials $g_V(t)$ and $g_H(t)$, are not constant. However, the variations of the coefficients are not totally random, as some could imagine. The strong correlation between points(close) that exists in the original trace should be maintained in the deformed trace.

This way, there should be a strong correlation between the values that $g_V(t)$ and $g_H(t)$ can assume, that is, considering the scale as an example, it is not allowed to utilise a scale factor for a point in the sequence defining the trace and, right after, do it for the subsequent point, another scale factor different from the previous one. This would cause a discontinuity of the deformed trace. The same thinking should be used in the definition of variation of the rotation angles.

The theory previously developed is extended for the case of non-constant coefficient and the conventions accepted are those previously described, plus the ones that follow:

- $g_H(t)$ and $g_V(t)$ are polynomials of order 0, deforms of horizontal and vertical scales, respectively, represented by functions of time;
- $g_H(t) = \dfrac{f_H(t) * \cos[\alpha_H(t)] + f_V(t) * \operatorname{sen}[\alpha_H(t)]}{f_H(t)}$ and

$$g_V(t) = \frac{-f_H(t) * \text{sen}[\alpha_V(t)] + f_V(t) * \cos[\alpha_V(t)]}{f_V(t)}$$ are polynomials of order 0, deforms of vertical and horizontal rotations, respectively, where $\alpha_H(t)$ and $\alpha_V(t)$ are functions of time representing the variations of the rotation angles; and

- D represents the duration of the tracing

3.4.1 - Random Variations of the Coefficients of the Deforming Polynomials

The deformations of traces through a deforming polynomial of order 0, with random coefficient, produces a trace with characteristics totally different from those of the original trace, because the strong relationship between close points is lost. Thus, the coefficient of a deforming polynomial of order 0 does not have a random variation, but varies according to some standard. This represents well what happens during the signatures writing. As the deforming polynomials represent disturbs occurring during writing, their coefficients cannot vary abruptly, because the disturbances does not vary abruptly.

3.4.2 - Standards of Variation of the Deforming Polynomials Coefficients

Considering that the more perceptible deformations occur in the trace extremes, a good representation of the variation of the deforming polynomials coefficients is through exponential.

Remembering that D represents the tracing duration, **Fig. 13** illustrates the exponential that can be used to define the intensity of the deforming polynomials coefficients at each time.

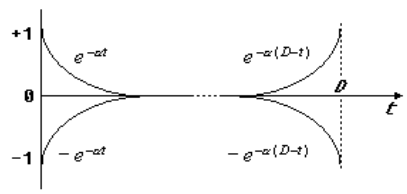

Fig. 13 . Deforming polynomials coefficients at each time

It is noticed that if $t=0$, at the beginning of the tracing, $e^{-\alpha t}=1$ and $e^{-\alpha(D-t)}$ tends to zero and, if $t=D$, end of the tracing, $e^{-\alpha t}$ tends to zero and $e^{-\alpha(D-t)}=1$. The constant α, positive, represents the speed with witch the exponential $e^{-\alpha t}$ and $-e^{-\alpha(D-t)}$ tend to zero, as well as the speed with witch the exponential $e^{-\alpha(D-t)}$ and $-e^{-\alpha t}$ tend to 1. The bigger the value of α, the faster the tendencies occur. Practically, reasonable values of α are less than 0.1.

The exponential previously described produce an undesirable collateral effect, in case they are utilised in the scale deforming polynomials.

It is noticed, for instance, that if the coefficient in a scale deforming polynomial of order 0 is given by $e^{-\alpha(D-t)}$, then the scale factors utilised for the deformation of the first points of the trace, depending on α and D, tend to zero. This would concentrate these points in a vertical axis that passes through the mass centre of the trace. That is not what is intended. The use of exponential functions that satisfy the limits

$$\lim_{t\to D} f_1(\pm e^{-\alpha t}) = 1 \text{ and } \lim_{t\to 0} f_2\left(\pm e^{-\alpha(D-t)}\right) = 1,$$

minimise those collateral effects present in the polynomials of order 0 that deform, in scale, the points of a trace.

For example, the functions : $f_1(e^{-\alpha t}) = \left(\frac{2+e^{-\alpha * t}}{2}\right)$ e $f_2\left(e^{-\alpha(D-t)}\right) = \left(\frac{2+e^{-\alpha*(D-t)}}{2}\right)$

produce interesting results, as shown in **Fig. 14**.

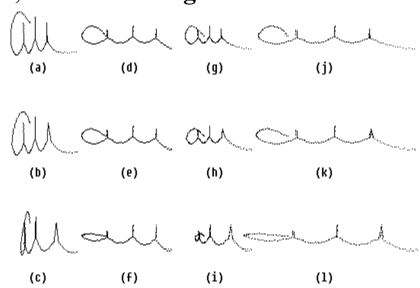

Fig. 14 . Interesting results

3.4.3 - Rotation

In the non-uniform rotation, the function for variation of the coefficients of the deforming polynomial does not have the collateral effects previously described in the non-uniform scale deformation.

Thus, any continuous function can be used for the variation of those coefficients. For instance, in Fig. 15, the variation of the rotation angle is uniform and is of 0°, in the beginning of the trace, for 30°, at the end of the trace.

3.4.4 - Scale and rotation

The deformations of non-uniform scale and rotation can be obtained starting from just one deforming polynomial. **Fig. 16** illustrates some combined deformations of non-uniform scale and rotation.

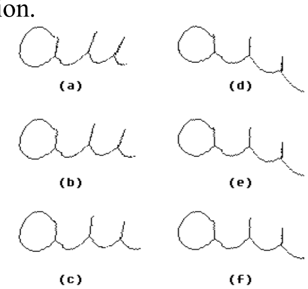

Fig. 15 . The variation of the rotation angle

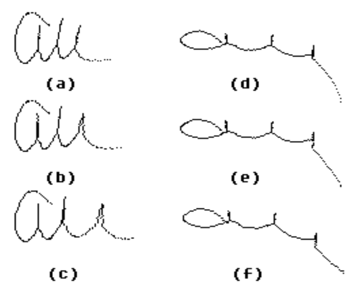

Fig. 16 . Some combined deformations of non-uniform scale and rotation.

3.5 - Deformations through instruments of degree N, N>0.

In the deforming polynomials of order 0, the deformation of a point depends only on the original position of the point itself and of the coefficient which represents the deforming polynomial. The correlation between the deformed point and its neighbours must be assured by the pattern of deformation of the deforming polynomial. In the deforming polynomials of order greater than 0, the deformation of a point also depends on the original position of its neighbours. Thus, the correlation between close points is more easily obtained, as may be seen on **Fig. 17**. The deforming polynomial used for deformation of the horizontal scale of the trace of the letters AU at **Fig. 17**, has order 6 and it is given by:

$$g_H(t) = \frac{c_6 t^6 + c_5 t^5 + c_4 t^4 + c_3 t^3 + c_2 t^2 + c_1 t^1 + c_0}{c_6 + c_5 + c_4 + 2 + c_2 + c_1 + c_0}$$

Fig. 17 . The correlation between close points.

where $c_0 = c_6 = 1$, $c_1 = c_5 = 2$, $c_2 = c_4 = 3$, e, c_3 is an aleatory value between 0 and 4. It is clearly noticed in Figure 17 that, although the coefficient *c3* tends to randomly distribute the points of the trace, the homogeneity of the other coefficients tends to maintain the correlation between the points. The choice of this polynomial, more specifically, of coefficient *c3*, emphasises the concept being discussed. Actually, possibly, none of the coefficients will be aleatory but functions that can be differentiated.

Continuing to talk about the previous polynomial, representing it as a sequence of values, it is obtained <1/14, 2/14, 3/14, c_3/14, 3/14, 2/14, 1/14> where *c3* varies from point to point according to its definition. Considering that (x_{t0}, y_{t0}) are the co-ordinates of the point $Pt'(t_0)$ of a trace T, then, the utilisation of that deforming polynomial for the deformation of the trace T makes the point $Pt'(t_0)$ to be moved for a point between the co-ordinates given by:

$$\left(\frac{x_{t-3}*1+x_{t-2}*2+x_{t-1}*3+x_{t0}*0+x_{t1}*3+x_{t2}*2+x_{t3}*1}{14}, y_{t0}\right)$$

$$\left(\frac{x_{t-3}*1+x_{t-2}*2+x_{t-1}*3+x_{t0}*4+x_{t1}*3+x_{t2}*2+x_{t3}*1}{14}, y_{t0}\right)$$

where the points $Pt'(t_{t-3})=(x_{t-3}, y_{t-3})$, $Pt'(t_{t-2})=(x_{t-2}, y_{t-2})$, $Pt'(t_{t-1})=(x_{t-1}, y_{t-1})$, $Pt'(t_{t1})=(x_{t1}, y_{t1})$, $Pt'(t_{t2})=(x_{t2}, y_{t2})$ e, $Pt'(t_{t3})=(x_{t3}, y_{t3})$ are the closest neighbours of $Pt'(t_{0})$. The previous equations emphasise the dependency that the deformation of a point has relative to the position of its n neighbours , where n is the order of the deforming polynomial**,** and with what intensity these dependencies occur, this given by the coefficients of the terms of the deforming polynomial relative the neighbour points.

An example of the use of deforming polynomials of order bigger than 0 is given below:

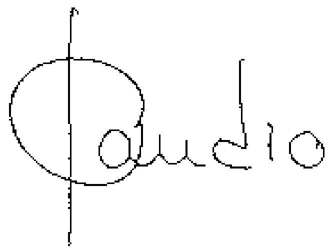

Fig. 18 . Original signature

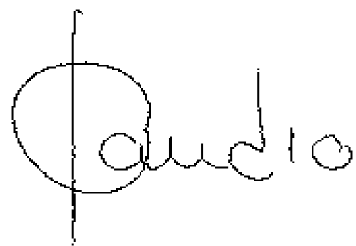

Fig. 19 . Deformations signature

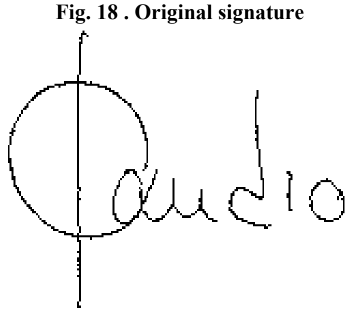

Fig. 20 . Deformations signature

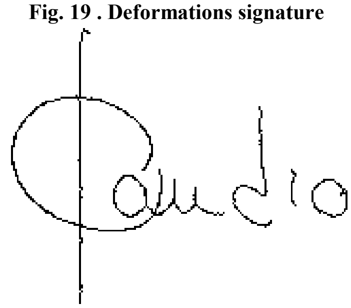

Fig. 21 . Deformations signature

Fig. 18, **Fig. 19** and **Fig. 20** show three deformations of the original signature, on **Fig. 21**. The deforming signatures were obtained starting from the original signature through deforming polynomials of order 5.

The polynomials for generation of the three deformations produce non-uniform scale deformations in both directions, horizontal and vertical, and are given by:

$$h_H(t) = 0.01 * c_{H_1}(t) * t + 0.05 * c_{H_2}(t) * t^2 + \left[1 + 0.1 * c_{H_3}(t)\right] * t^3 + 0.05 * c_{H_4}(t) * t^4 + 0.01 * c_{H_5}(t) * t^5$$

$$h_V(t) = 0.01 * c_{V_1}(t) * t + 0.05 * c_{V_2}(t) * t^2 + \left[1 + 0.1 * c_{V_3}(t)\right] * t^3 + 0.05 * c_{V_4}(t) * t^4 + 0.01 * c_{V_5}(t) * t^5$$

where each term $c_{Hi}(t)$ and $c_{Vi}(t)$ is a differentiable function. For each trace of the original signature it was used an independent pair of deforming polynomials, that is, dependencies between deformation of different traces were not considered.

The functions $c_{Hi}(t)$ and $c_{Vi}(t)$ were obtained through the Gregory-Newton's interpolating polynomial, starting from 6 aleatory values between -1 and +1, homogeneously distributed in the duration time of the corresponding traces. **Fig. 22** illustrates the interpolation of the values 0.36, -0.19, 0.80, -0.56, 0.84 and, -0.82 in the corresponding times, 0, 62, 124, 186, 248 e, 310, being D=310. It can be seen on Figure 22 that, due to the interpolation $f(t)$ is known for every t complying with $0 \leq t \leq D$.

As the values of $c_{Hi}(t)$ and $c_{Vi}(t)$ should be known at any time between the beginning and the end of the tracing, the distribution of the aleatory points in the time of the duration of all the tracing is justified.

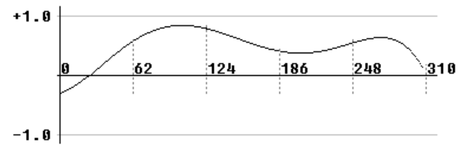

Fig. 22 . The interpolation of the values 0.36, -0.19, 0.80, -0.56, 0.84 and, -0.82

The limits in which the values for interpolation can be randomly selected, -1 and +1, allow an easy manipulation of the intensities with which each term of the deforming polynomial contributes for the total deformation. For instance, the constants 0.01, 0.05 and 0.1 when used in the interpolations, limit the contributions of each term of the deforming polynomial. Finally, the addition of the constant 1 to the coefficient $1+0.1 * c_{H_3}(t)$ of t^3 defines a stronger tendency to the maintenance of the tracing. Isolately interpreting this term, it can be said that the constant 1 represents the trace without deformation and $0.1 * c_{H_3}(t)$, with $-1 \leq c_{H_3}(t) \leq +1$, a tendency of about 10% to deformation.

4. CONCLUSION

In a first important observation, we can verify that, if each variable interfering in the process of the writing of the signature traces is represented under the form of a coefficient of a deforming polynomial, then, this polynomial has the power of pondering the interference's of each of those variables, in order to produce deformations, at least, very close to those which occur in real situations. It is also important to observe that as there are variables tending to the deformation of the tracing, there are also variables which tend to keep the tracing - these, inclusively, having a bigger contribution than the former. Thus, for any interpretation of the variables interfering in the process of writing of traces, the use of an adequate deforming polynomial, or the composition of various polynomials, as previously described, has the power of capturing any deformation of tracing wanted, from the model of traces of an original signature.

The results so far obtained are promising and can be used for future works with much reliability.

BIBLIOGRAPHIC REFERENCES

[DOE93] – Doermann, D. S. & Rosenfeld, A., «The Interpretation and Reconstruction of Interfering Strokes», In Frontiers in Handwriting Recognition III, pages 41-50, CEDAR, SUNY Buffalo, New York, USA, May 25-27, 1993.

[PLAM89] – Plamondon R, Lorette G – «Automatic Signature Verification and Writer Identification – the State of the Art «, Pattern Recognition, vol. 22, nº 2, pag. 107 a 131, 1989.

[PLAM90] – Plamondon R, Lorette G, Sabourin R – «Automatic Processing of Signatures Images : Static Techniques and Methods «, Handwritten Pattern Recognition, 1990.

[RAND90] – Randolph D, Krishnan G – «Off-Line Machine Recognition of Forgeries» – Machine Vision Systems Integration in Industry, vol. 1386, pp. 225 a 264, 1990.

[SABO90] – Sabourin R, Plamondon R, Lorette G – «Off-Line Identification with Hand-written Signature Images : Survey and Perspectives», SSPR, 1990.