Throughput Analysis Based on One Hop transmission in Wireless Ad hoc Networks

K. Mnif B. Rong and M. Kadoch
LAGRIT, Ecole de Technologie Supérieure
1100, Notre Dame O.
Montreal, Qc, CANADA
kais.mnif.1@ens.etsmtl.ca

Abstract

The main concerns of network performance are throughput, end-to-end packet delay and packet loss. A high-performance network is characterized by high throughput, small delay, and low packet loss. In this paper, we introduce a theoretical model to analyse the one hop throughput with regular structure operating under the CSMA/CA access protocol in wireless ad hoc networks. The numerical results of this analysis and the simulation results show that we can find an optimal transmission range to achieve the highest throughput. An optimal transmission range is essential to limit the energy dissipation on the mobile devices.

***Keywords*:** wireless ad hoc networks, Collision Avoidance, throughput, transmission range

I. INTRODUCTION

With the increase of mobile computing devices, the demand or continuous network connectivity without physical infrastructure has encouraged the interest in the use of the wireless mobile ad hoc networks. An ad hoc network is a network in which a group of terminals communicate between themselves using wireless radios, without the support (help) of a fixed networking infrastructure.

Wireless ad hoc networks differ from cellular networks which can reuses frequency. Frequency reuse in mobile cellular systems means that each cell has a frequency that is far enough away from the frequency in the bordering cell that it does not provide interference problems. In ad hoc networks the maximum number of simultaneous transmissions supported by the network is a function of the network density, i.e. range transmission of each terminal.

In as ad hoc network, the bandwidth available to a node at any given time is limited and is affected by fading and shadowing effects in the wireless channel, changes in the network topology, and variations in the background traffic generated by the surrounding (neighbouring) nodes. These effects, which do not exist in wired networks, make the evaluation and the analysis of the throughput a challenging task. Particularly in wireless ad-hoc networks, characterised by dynamic topology and limited bandwidth. The optimal transmission power to maximize the expected progress involves the following compromises. A short-range transmission is preferred in terms of successful transmission because its avoid collision at the receiver. A long-transmission is favourable in term of: *i*- it moves a packet the maximum it can in one hop in successful transmission and *ii*- the probability to find a receiver candidate in the progression direction [1].

With multiple user channel access techniques such as carrier sense multiple access/collision avoidance (CSMA/CA), when omnidirectionnal antennas are used, neighbours of both the transmitter and the receiver *terminal*s must be blocked from transmitting on the same channel as transmitter/receiver [2]. The distribution coordination function (DCF) specifies the use of the CSMA protocol with CA. The CSMA used in wireless networks is similar to the CSMA scheme used in wired LAN. However, the Collision CA technique for wired LANs cannot be used effectively for wireless LANs since nodes cannot detect over the air occurred collisions. The CSMA/CA for wireless LANs allows for options that can minimize collisions by using RTS (request to send), CTS (clear to send), data and ACK acknowledge [3]. Neighbours nodes that overhear the RTS or CTS packets defer their access to the shared channel to avoid collisions. In fact, this collision avoidance scheme depends on the broadcast nature of the channel [4].

Throughput analysis is one of several important parameter of network performance; it attracts the attention of many researchers. In contrast to the transmission of small messages, new application can consume as much network bandwidth as is available. It is the end-to-end throughput rather than the per packet end-to-end delay that is the main performance concern [3]. Throughput can be defined as the maximum fraction of channel bandwidth used by successfully transmitted messages [5]. In [6], authors provide an analytic tool to evaluate the expected throughput of the route. Given a routing path, they determine the expected route throughput based on the random walk model. They assert that path throughput can be used as a critical route selection criterion into most of the current routing protocols for MANETs. In [7], we give an analysis of the maximum throughput of slotted–ALOHA–based multihop ad hoc networks with and without capture. An interesting analysis is showed when each node have a different transmission probability. Results show that the network throughput is greatly affected by the node degrees, both for capture and non-capture slotted ALOHA. Wang and Garcia-Luna-Aceves [4] present analysis of three collision-avoidance protocols that use omni-directional packet reception together with omni-directional transmissions, directional transmissions or a combination of both. Their analysis results show that collision avoidance using a narrow antenna beamwidth for transmission of all control and data packets achieves the highest throughput. Gkelias *et al.* [8] were derived for the throughput performance of multi-hop CSMA/CA system, as a function of the offered load,

Different from previous works, in this paper we give a detailed throughput analysis based on one hop transmission analysis in ad hoc networks using the CSMA/CA protocols at the physical layer.

This paper is organized as follow; section II gives a description of the network model, notations and assumptions used for this analysis. Section III presents our throughput analysis for one hop; our objective is to find the optimal conditions for maximum throughput. Section IV presents simulation results and compared to numerical results derived from the analytical model. Section V concludes this paper and proposes some directions for future work.

II. Network model and notations

In a given square area with edge L, n terminals are two-dimensionally Poisson distributed with parameter λ. Where λ is the distribution density of terminals per square units,

$$\lambda = n/L^2.$$

All terminals use the same transmission range, R. $Area(P,R)$ defines the coverage area or the transmission range where P is the center and R is the radii. Any node located in this area can hear what terminal P is sending. Let N be the number of terminals located in the $Area(P,R)$. $N = \lambda \cdot S$ and $S = \pi R^2$, then:

$$N = \lambda \pi R^2.$$

N defines also the average degree. Intuitevely, if there are more neighbours, i.e., higher node degree, the terminal should transmit less to avoid packet collision. Let p the probability that a terminal transmits, and $1-p$ the probability that it doesn't transmit. The transmission form P to Q, denoted by $P \rightarrow Q$.

III. One hope throughput analysis

In this section, our aim is to find optimal conditions (p_o, N_o) to maximize the throughput at one hope transmission. Let define $T_h(p,N)$ the one hop throughput, defined as the average number of successful transmissions from a terminal and $P_r(p,N)$ the expected progress of a packet in the direction of its final destination from a terminal.

We like to determine the expected number of successful transmissions per slot for an arbitrary terminal in the network. The probability that there is no terminal, except P, in the $Area(P,R)$ is equal to e^{-N}. In this case no transmission will be achieved. Let A_i be the event that there are i other terminals. Then, the probability that i terminals are located on the area S is given by:

$$p(A_i,S) = \frac{(\lambda S)^i}{i!} e^{-\lambda S}, \; i = 0, 1, 2, \ldots \quad (1)$$

Within the $Area(P,R)$, the average number of terminals is $N = \lambda \pi R^2$ So equation (1) can be written:

$$p(i,N) = \frac{N^i}{i!} e^{-N}, \; i = 0, 1, 2, \ldots \quad (2)$$

The probability, at the event A_i, that there is no terminal transmits is:

Prob. $\left[P \rightarrow Q \mid A_i\right] = (1-p)^i$, $i = 0, 1, 2, \ldots$

Transmissions $P \rightarrow Q$ will be successful, if neighbours terminals of transmitter and receiver situated in the shaded area, figure 1, are blocked for transmitting on the same interval of P.

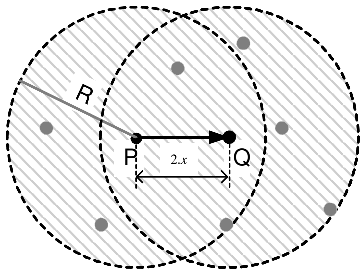

Figure 1 – None of terminals situated in the shaded area should transmit when P transmits

The grey zone is a function of the distance between P and Q and it's defined by the following equation [9]:

$$f(x) = 2\pi R^2 \left(1 - q(x)/\pi\right)$$

$$\text{where} \quad q(x) = \cos^{-1}\left(\frac{x}{R}\right) - \frac{x}{R}\sqrt{1-\left(\frac{x}{R}\right)^2} \qquad (3)$$

The distance between P and Q is equal to $2x$.

In the first step, we start by analysing the throughput at one hope, T_h, and find its expression as a function of p and N. Our objective is to find a relation between p and N to maximize one hope throughput. As mentioned before one hope throughput is the average number of successful transmission can be achieved by one terminal. To compute the average of successful transmission by terminal, P, the set of the following probabilities should be true

p_1; prob.[there exist at least one terminal, Q, in the $Area(P,R)$],

p_2; prob.[P transmit],

p_3; prob.[Q doesn't transmit],

p_4; prob.[no terminal in the $Area(P,R)$ and $Area(Q,R)$ transmit in the same interval of P].

$Th_{(p,N)} = p_1.p_2.p_3.p_4$

Where $p_1 = 1 - e^{-N}$, $p_2 = p$, $p_3 = 1 - p$ and

$$p_4(x) = \sum_{i} \text{Prob.}\left[P \to Q \mid A_i\right].p\left(i, \lambda f(x)\right)$$

$$= \sum_{i=0}^{\infty} (1-p)^i \times p\left(i, \lambda f(x)\right) = e^{-2pN(1-q(x)/\pi)}$$

$$T_{h(p,N)} = p(1-p)(1-e^{-N})g(N,p) \qquad (4)$$

where $g(N,p) = \int_{x=0}^{x=1/2} e^{-2pN(1-q(x)/\pi)}dx$, $R = 1$

$q(x)$ is given by equation (3).

By inspecting the waveform of $g(N,p)$, it can be approximated to an incremental exponential function of the product pN. This allows further simplification on the computation of $Th_{(p,N)}$, therefore a simple expression of $g(N,p)$ can be given by :

$$\hat{g}(N,p) = \frac{1}{2}e^{-1,295\,p.N}$$

Figure 2 shows the curve of $g(N,p)$, evaluated by Simpson method 3/8 [10], and its approximation $\hat{g}(N,p)$.

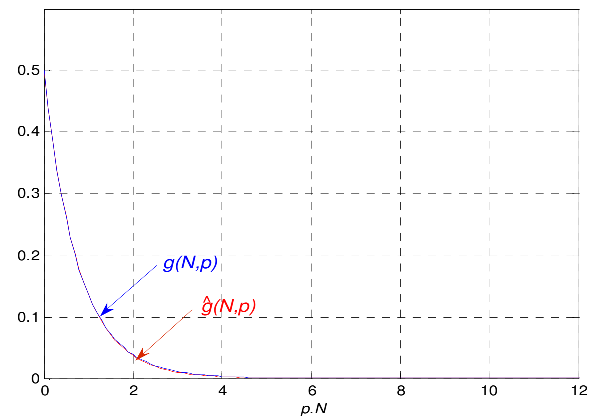

Figure 2 – $g(N,p)$ can be approximate by $\hat{g}(N,p) = \frac{1}{2}e^{-1,295p.N}$ for $pN \geq 0$

Let now finding the expression of $p_o = p(N)$ which maximizing $T_{h(p,N)}$

$$\frac{\partial Th(p,N)}{\partial p} = 0 \Rightarrow p_o(N) = \frac{N + 1.55 - \sqrt{N^2 + 1.55^2}}{2N} \qquad (5)$$

We can verify that we have $0 < p_o < 1$ foe all $N \geq 0$ by observing the limit; $\lim_{N=0} p_o(N) = \frac{1}{2}$ and $\lim_{N=\infty} p_o(N) = \frac{1}{N} = 0$, $0 < p < 1$ for all $N \geq 0$. We reject the second root because it doesn't satisfy the condition $0 < p < 1$.

The waveform in figure 3 shows p_o versus the average number of neighboor, N. For the given expression of $p_o(N)$, equation (5), the average throughput, equation (4), is

$$T_h(p_o(N),N) = \frac{1}{2}p_o(N)(1 - p_o(N))(1 - e^{-N})\, e^{-1.295Np_o}$$

when $N \to \infty$, we have $\lim_{N \to \infty} p_o(N) = \frac{1}{N}$,

then $\lim_{N \to \infty} T_h(p_o(N),N) = \frac{1}{N} = 0$.

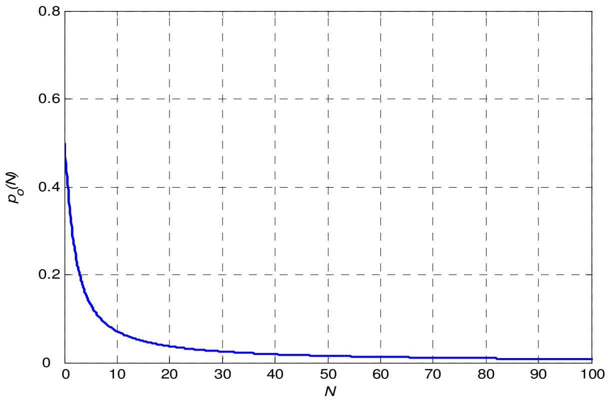

Figure 3 – Transmission probability as a function of N

When $N \to 0$, only 2 terminals P and Q are located on the $Area(P,Q)$ and they can transmit with probability equal to $\frac{1}{2}$ (the well known optimal p for two terminals). The optimized results for large N are the same in [1].

Figure 4 shows the curve of T_h as a function of N.

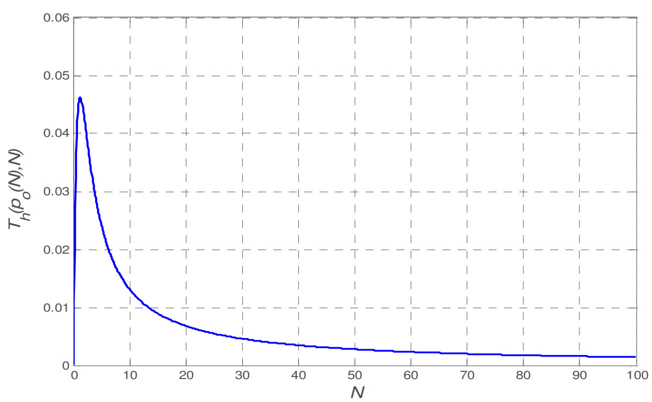

Figure 4 – T_h as a function of N

The second step consists on the analysis of the expected progress. As in the first step, our objective is to maximize the expected progress, so we went to find conditions for p and N to achieve this.

In order to analysis the expected progress, $P_r(N,p)$ of a packet in the direction of its final direction from the transmitter. The progress z is attained where z is he distance between the transmitter and the receiver projected onto the axis source-destination, figure 5.

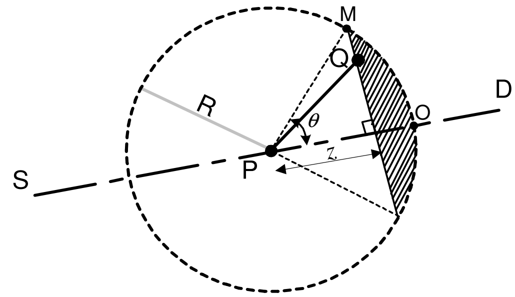

Figure 5 – The expected progress, z, from P to Q.

The shaded area is defined by

$f(x) = \cos^{-1} x - x.\sqrt{1-x^2}$ for normalized R=1,

$P_r(p,N) = p_1 \times p_2 \times p_3 \times E$[the packet progresses]

p_1; prob.[P transmits],
p_2; prob.[Q doesn't transmit],
p_3; prob.[no terminal in the $Area(P,R)$ and $Area(Q,R)$ transmits in the same interval of P]

where $p_1 = p$,
$p_2 = 1 - p$,
$p_3 = \sum_i \text{Prob.}[P \to Q \mid A_i].p(i, \lambda f(z))$,

where $p(i, \lambda f(z)) = \frac{(\lambda f(z))^i}{i!} e^{-\lambda f(z)}, i = 0, 1, 2, \ldots$

E[the packet progresses]$= \int_{-R}^{R} z.\text{Prob}[z < \tilde{y} \le z + dz]$

Then

$$P_r(p,N) = p(1-p)\hat{g}(N,p) \int_{-R}^{R} z.\text{Prob}[z < \tilde{y} \le z + dz] \quad (6)$$

here $\hat{g}(N,p) = \frac{1}{R^2} e^{-1{,}295\, p.N}$

The integration $\int_{-R} z.\text{Prob}[\ z < \tilde{y} \le z + dz\ ]$ is a function of N only and do not depend of p.

so $P_r(p,N) = p(1-p)\hat{g}(N,p)h(N)$,

where $h(N) = \int_{-R}^{R} z.\text{Prob}[\ z < \tilde{y} \le z + dz\ ]$

and we have $\text{Prob}[\tilde{y} \le x] = e^{-\frac{N}{\pi}(\cos^{-1} z - z.\sqrt{1-z})}$

$h(N)$ can be expressed by [1] :

$$h(N) = \sqrt{\frac{N}{\lambda\pi}} \left[1 + e^{-N} - \int_{-1}^{1} e^{-\frac{N}{\pi}(\cos^{-1} z - z.\sqrt{1-z})} dz \right] \quad (7)$$

for R=1

p_o, from equation (5), maximise also $P_r(p,N)$ and we have $\lim_{N=0} p_o(N) = \frac{1}{2}$ et $\lim_{N\to\infty} p_o(N) = \frac{1}{N}$,

$P_r(p_o(N),N)\sqrt{\lambda}$ =

$$p_o(N).(1-p_o(N)).\sqrt{\frac{N}{\pi}}\left[1+e^{-N}-\int_{-1}^{1} e^{-\frac{N}{\pi}(\cos^{-1} z - z.\sqrt{1-z})} dz\right]$$

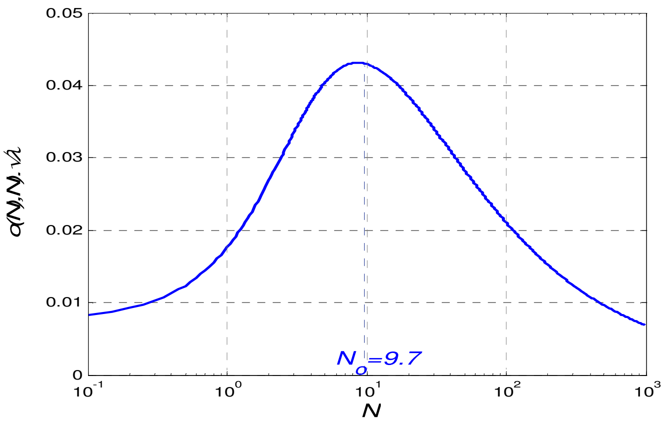

Figure 6 – $P_r(p_o(N),N)\sqrt{\lambda}$ as a function of N

Using a numerical integration method [Gerald and Wheatley '03] to evaluate $h(N)$, we can get,

$$h(0) = 0 \text{ et } h(\infty) = \sqrt{\frac{N}{\lambda\pi}}$$

and therefore $\lim_{N=0} P_r(p_o(N),N)\sqrt{\lambda} = 0$ et $\lim_{N=\infty} P_r(p_o(N),N)\sqrt{\lambda} = 0$, P_r is positive for all $N \geq 0$ so the optimal value of T_h can be founded for $N=N_o$. We are looking to find this number N_o. Due to the complexity of the $P_r(p_o(N),N)$ expression, we determine N_o using the graphical method.

Figure 6 shows the curve of $P_r(p_o(N),N)$ and the N_o can easily determined, it s located between 9 and 10.

We find that $P_r(p_o(N=9),N=9)\sqrt{\lambda} > P_r(p_o(N=10),N=10)\sqrt{\lambda}$ then we choose N_o=9, so 9 represents the optimal value for the number of neighbours, to get a maximum progression of the packet on the direction of its destination.

III. Simulation results

We consider the case of finite networks. In the finite networks, we consider a square LxL as area of the network and we randomly place n terminals in the square. Our simulation environment contains n=200 nodes distributed in a square area with edge L=200m, so $\lambda = n/L^2$ = 2,5 10^{-3} nodes/m^2. Nodes are placed in this area with the exponential distribution, parameterλ. According to the analysis of the previous section, the optimal number of neighbour is N_o=9, or $N=\lambda\pi R^2$, then we deduct the optimal range transmission R_o = 23,9m.

We are interested to the variation of the throughput versus the transmission probability. With a fixed transmission range, R_o=23.9m, we except to get a number of neighbour equal to N_o=9, and we vary p from 0 to 1. To reduce the negative effects induced by the ''border effect'', the nodes close to the edges of the square area which have smaller number of neighbours, we consider only nodes located Analysis results show that the optimal value, for maximum throughput, is p_o=0.08. For the experimental results, curve from figure 7 shows that the throughput is a little bit higher than the analysis one, this can be explained by; nodes are plotted randomly on the square area, with a fixed transmission range, we expect to get a number of neighbours equal to 9 for each terminal, or due to simulation concern, the average number of neighbour is N^{exp}=8.8, which is smaller than the expected value.

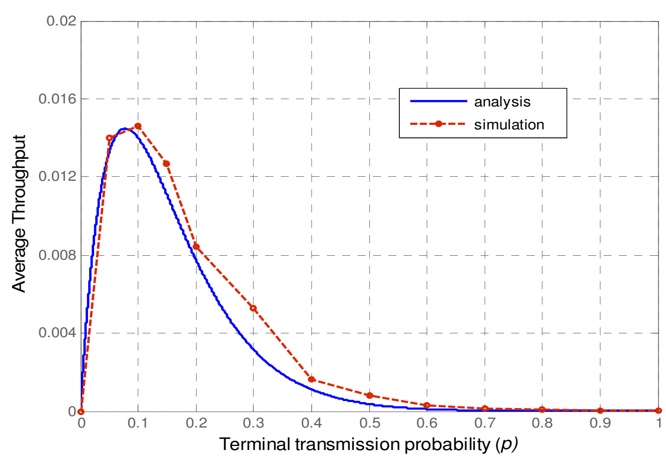

Figure 7 – Average throughput over transmission probability (p)

Table 1 illustrates multiple simulation runs; for the same parameters as above, i.e. number of nodes and area size, we compute the average and the variance of the number of neighbour and the optimal transmission probability. Results from this table show that when the average number of neighbour is greater (resp. smaller) than the optimal value, we get an optimal transmission smaller (resp. greater) than the optimal one.

Table 1: Multiple simulations run

Simulation #	Number of neighbour		Optimal transmission probability
	Average	variance	
1	8,80	0,22	0,10
2	8,65	0,19	0,12
3	9,18	0,10	0,07
4	8,68	0,23	0,11
5	9,16	0,17	0,07
6	8,75	0,28	0,12
7	8,91	0,11	0,08
8	8,74	0,24	0,11
9	9,23	0,25	0,07
10	8,52	0,34	0,13

IV- CONCLUSION

In this paper we developed an analytical model for the throughput performance for one hope transmission with CSMA/CA system. We formulate the throughput as a function of number of neighbour and range transmission. These results can be used as a guide for further throughput and delay analysis on CSMA based MAC protocols. Our future work will include the throughput on a route for multi-hop and also it will be interested to evaluate the throughput of the whole network.

References:

[1] Takagi, H. and Kleinrock L. "Optimal Transmission ranges for randomly distributed packet radio terminals." IEEE Transactions on communications, vol. 32 (3), pp. 246–257. March 1984.

[2] Aggelou, G. *Mobile Ad hoc Networks: from wireless LANs to 4G networks*, McGraw-Hill. 2005.

[3] Futernik, A., Haimovich A. M. and Papavassiliou S. "An analytical Model for Measuring QoS in Ad hoc Wireless Networks." Globecom, vol. 8 (3), San Francisco, CA, USA, pp. 216–220. December 2003.

[4] Wang, Y. and J. J. Garcia-Luna-Aceves "Collision Avoidance in Single-Channel Ad hoc Networks Using Directional Antennas." International Conference on Distributed Computing Systems'03, vol. 3, pp. 1788–1794. Rhode Island USA, May 19–22, 2003.

[5] Conti, M., Geogri E. and Lenzini L. (2002). "Mobile Ad hoc Networking." Clustering Computing Journal, vol. 5 (2).

[6] Tseng, Y.-C., Chu W., Chen L.-W. and Yu C. M. "Route Throughput Analysis for Mobile Multi-RateWireless Ad Hoc Networks." First International Conference on Broadband Networks (BROADNETS'04), pp. 469–475, 25–29 October 2004, San Jose, CA 2004.

[7] Yao, Z., Li V. O. K. and Cao Z. "Maximum Throughput Analysis and Enhancement of Slotted ALOHA for Multihop Ad Hoc Networks." IEEE International Conference on Communications vol. 7, pp. 4162–4166, 20–24 Juin 2004, Paris 2004.

[8] Gkelias A. Dohler M. and Aghvami H. "Throughput Analysis for wireless Multi-hop CSMA" 15th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, 2004. PIMRC'04 vol. 2, Page(s):979 – 983. 5-8 Sept. 2004.

[9] Spiegel, M.R. (ed.) *Mathematical Handbook of Formulas and Tables*, McGraw-Hill, New York. 1968.

[10] Gerald, C. F. and Wheatley P. O. *Applied Numerical Analysis*, Addison-Wesley, Boston, 2003.