Performance Evaluation Of Tree-based Reliable Multicast

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Abstract--**The expected number of retransmissions is a very important parameter to evaluate the multicast performance. It is often used to estimate other performance parameters such as the bandwidth consumption and delay. A simple and efficient evaluation to these parameters is helpful for design of multicast protocols. In this work, we will derive a good analytical evaluation to the number of retransmissions. We also obtain the total bandwidth consumption for hierarchical reliable multicast where some special receivers or routers are assigned for retransmission handling of each domain. The optimal partitioning of the tree also depends on topology of networks for each subgroup. The effects of topology on optimal placements are analyzed using bandwidth consumption as criteria for hierarchical reliable multicast[1].**

Keywords — **reliable multicast, topology, optimization.**

1. Introduction

A number of reliable multicast protocols have been proposed in recent years[1][3]. Performance analysis of basic parameters such as delay and bandwidth consumption has been compared for different kinds of protocols in some references [12][11][5][7]. For reliable multicast, loss recovery is crucial. Each lost packet has to be retransmitted to these receivers who didn't get the packet in previous transmission and retransmissions. For analysis of bandwidth and delay, one often uses one important parameter to calculate the retransmitting times, that is, the expected number of retransmissions for one packet[2][4][5][11]. For reliable multicast, delay and bandwidth will greatly depend on the number of retransmissions. They also depend on other factors such as loss recovery, loss characteristic, network topology and partitioning of networks. Evaluations to these parameters are very important criteria to reliable multicast.

Many analysis models don't consider loss from intermediate links, i.e, loss free intermediate model. Bandwidth is wasted mainly by lossy receiver links whose topology is star. Impacts of topology are neglected in this case. However, these intermediate routers may lose the packets of the multicast network, and losses may still exist in intermediate links. For example, due to overflow of buffers of intermediate routers, packets may be lost. Loss from an intermediate link will result in retransmissions to many links and consume more bandwidth. Impact of these losses from intermediate links on the multicast network is significant. In the old model of loss-free intermediate links, effects of topology can't be estimated. If intermediate links are considered to be lossy, one has to consider influence of topology. Even if they have the same number of links, networks of different topologies may have different the number of retransmissions, then have different bandwidth consumption.

In this work, we will focus on effects of topology on multicast performance. We will derive a good analytical evaluation to the number of retransmissions so that the sender or repair servers can easily evaluate it, then apply this result to choice of locations of repair servers based on lossy model of intermediate links.

2. Performance Analysis

2.1 Previous Work

The expected number of transmission and retransmissions E[M] that a packet should be multicast by the source until all group members receive it correctly can be recursively calculated [4]. For a multicast network, one can calculate the CDF (cumulative distribution function) of the total number of transmissions and retransmissions from the sender. Let M(n) be the total number of transmission and retransmissions of a packet until received by all receivers under node n. Then CDF for node n is $F_n(m)=P[M(n)\le m]$, i.e, $F_n(m)$=Prob.[all nodes from n and below got the packet at most in m trials]. We denote $F_r(m)$, $F_n(m)$ and $F_s(m)$ to be CDF of the total number of transmission and retransmissions for leaf receivers (r), nodes (n) and the sender (s), respectively. Then one can obtain the following equations for E[M] [4] [5].

$$E[M(S)] = \sum_{m=o}^{\infty} mP_S(M=m) = \sum_{m=o}^{\infty}(1-F_s(m)) \qquad (1)$$

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$$F_S(m) = \prod_{c \in child(S)} F_c(m) \qquad (2)$$

$$F_n(m) = \sum_{u=0}^{m-1} \binom{m}{u} p_n^u (1-p_n)^{m-u} \prod_{c \in child(n)} F_c(m-u) \qquad (3)$$

$$F_r(m) = 1 - p_r^m \qquad (4)$$

where p_n or p_r is packet loss probability on the link leading to node n or leaf receiver r.

Needless to say one should use recursion starting from bottom of nodes and numerical evaluation of equation (1)-(4) to calculate E[M]. In this paper, we follow a completely analytical approach for evaluation of E[M] without needing recursion. For analysis convenience, we use homogeneous packet loss probability in the following, that is, all links have the same packet loss probability p.

2.2 Probability Of The Number Of Transmissions

For tree-based reliable multicast, packet loss of one intermediate node will result in retransmissions to many nodes. Once one intermediate node loses the packet, all the nodes under the node will lose the packet. For example, if node k_1 loses the packet, n_{k_1} nodes won't receive the packet where n_{k_1} is the number of links under the node k_1. In the next retransmission, we only care whether n_{k_1} nodes receive the packet or not. Unavoidably, one has to pass through the path from the sender to node k_1 to recover loss. The number of nodes to be retransmitted depends not only on the number of nodes that didn't receive the packet but also the number of nodes within the path from the sender to node k_1. Therefore, we only need to care that these nodes receive the packet correctly in the next retransmission.

Table 1 Notations for reliable multicast

a.	N is the total number of links.
b.	p is packet loss probability of each link.
c.	M, m is the number of transmission and retransmissions for one packet.
d.	d_k is the number of links from the sender to the certain node (node k).
e.	n_k is the total number of links under the node k.
f.	$d_{k_1 k_2 \cdots k_i}$ is the total number of links from the sender to node $k_1, k_2, \ldots, k_i$, i=1,2,…
g.	P(M=m) is probability of m transmission and retransmissions for one packet.
h.	P(M=m|$k_1 k_2$…) is conditional probability of m retransmissions after node k_1, node k_2, ..., lose the packet in previous transmission.

If some losses take place again out of n_{k_1} nodes in the next transmission, we need to repeat the above process until all nodes receive correctly the packet. For the other nodes that have received correctly the packet, we no longer care if they receive the packet again. Therefore, each multicast retransmission is aimed to recover losses from previous transmission or retransmissions. The first original transmission of data packet may be considered to recover losses from all nodes because all nodes don't have the packet.

After we understand the process of loss recovery, we can calculate the performance of multicast transmissions and retransmissions. For each retransmission, each link possibly loses the packet. For losses from different nodes, the sender needs different retransmission times. Therefore, probability density of the number of transmissions m depends on which nodes lose the packet.

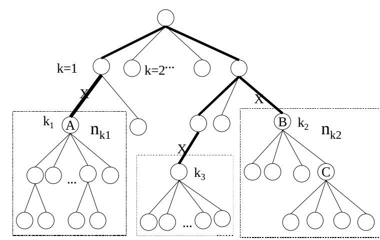

Fig. 1 One example of general tree

Here, we first need to distinguish the kinds of losses. If loss of one link isn't under subtree of another link loss, two losses don't bother each other. They are independent. For example, losses of node k_1 (A) and k_2 (B) are independent. However, if one link is under subtree of other link loss, these losses are dependent. For example, losses of node B and C are dependent because loss of node B definitely results in loss of node C. In the following, loss means independent loss that don't bother each other.

a. No loss occurs

Probability that no loss occurs is

$$\text{Prob.(no loss)} = (1-p)^N \qquad (5)$$

b. Only one loss occurs

Assume only one loss occurs, for example, node k_1 lost the packet. Probability that only node k_1 lost the packet is

$$\text{Prob.(only node } k_1 \text{ loses)} = p(1-p)^{N-n_{k_1}-1} \qquad (6)$$

where n_{k1} is the number of links under node k_1.

c. j losses occur

Probability that node k_1, k_2,…, k_j lost the packet in one transmission is

$$\text{Prob.(node } k_1, k_2 \ldots k_j \text{ lose)} =$$

$$\begin{cases} p^j (1-p)^{N-\sum_{s=1}^{j} n_{k_s} - j}, & k_2 \notin D_{k_1}, \cdots, k_j \notin D_{k_1 k_2 \cdots k_{j-1}} \\ 0, & otherwise \end{cases} \qquad (7)$$

where $D_{k_1 k_2 \cdots k_i} = d_{k_1 k_2 \cdots k_i} + \sum_{s=1}^{i} n_{k_s}$ (i=1,2,…,j), n_{k_s} is the number of links under node k_s (s=1,2,…,N).

$d_{k_1k_2\cdots k_i}$ is the total number of links from the sender to node $k_1,k_2,\ldots,k_i$, for example, $d_{k_1k_2}$ =4 (in bold), $d_{k_2k_3}=4$, $d_{k_1k_2k_3}=6$ in Fig. 1.

After we know probability that nodes lose the packet in one transmission, we can recursively calculate the probability density of the number of transmissions m. Probability of only one transmission is obvious.

$$P(M=1)=(1-p)^N \tag{8}$$

If some nodes lose the packet in the first transmission and the sender can recover these losses in the second retransmission, one can obtain the probability of two transmissions.

$$\begin{aligned}P(M=2)&=\sum_{k_1=1}^{N}p(1-p)^{N-n_{k_1}-1}P(M=1\,|\,k_1)\\&+\frac{1}{2!}\sum_{k_1=1}^{N}\sum_{k_2\notin D_{k_1}}p^2(1-p)^{N-n_{k_1}-n_{k_2}-2}P(M=1\,|\,k_1k_2)+\cdots\end{aligned} \tag{9}$$

where $P(M=1|k_1k_{2\ldots}k_s)$ is conditional probability that 1 retransmission is needed to make multicast successful if node k_1, $k_{2\ldots}k_s$ lose the packet in the first transmission.

Similarly we can obtain the probability of m transmissions.

$$\begin{aligned}P(M=m)&=\sum_{k_1=1}^{N}p(1-p)^{N-n_{k_1}-1}P(M=m-1\,|\,k_1)+\cdots\\&+\frac{1}{j!}\sum_{k_1=1}^{N}\sum_{k_2\notin D_{k_1}}\cdots\sum_{k_j\notin D_{k_1\cdots k_{j-1}}}p^j(1-p)^{N-\sum_{s=1}^{j}n_{ks}-j}P(M=m-1\,|\,k_1k_2\cdots k_j)+\cdots\end{aligned} \tag{10}$$

where $P(M=m-1|k_1k_2\ldots k_j)$ is conditional probability of m-1 retransmissions after node k_1, node k_2, ..., k_j lose the packet in previous transmission, for example, $P(M=m-1|k_1)$ is conditional probability of m-1 retransmissions if node k_1 loses the packet.

2.3 Analytical Approximation Of E[M]

For each subgroup in multicast network, N>>1, p<<1, Np<1. If Np>1, the sender needs to multicast the packet so many times that the protocol can't work properly. Thus, we need to design loss recovery to reduce the number of retransmissions. Local recovery and partitioning a group can be efficiently used to reduce range of loss recovery and further reduce the number of retransmissions. Therefore, we only consider the case of Np<1 here and that assumption is very reasonable.

For a multicast subgroup of Np<1, we may expand the above probability density function of M according to the order of loss probability p. In the following, we will consider till the 3rd order approximation of p or Np. From (8), one may have

$$P(M=1)=(1-p)^N=1-Np+\binom{N}{2}p^2-\binom{N}{3}p^3+\cdots \tag{11}$$

In order to calculate probability of two transmissions P(M=2), one needs to only retransmit one time after losses take place. For example, if only node k_1 loses the packet in the first transmission, probability that the next retransmission is successful is $P(M=1\,|\,k_1)=(1-p)^{d_{k_1}+n_{k_1}}$ where d_{k_1} is the number of links from the sender to node k_1 and n_{k1} is the number of links under node k_1. Similarly, one can obtain

$$\begin{aligned}&P(M=1\,|\,k_1k_2\cdots k_j)=(1-p)^{d_{k_1k_2\cdots k_j}+\sum_{s=1}^{j}n_{k_s}},\\&k_2\notin d_{k_1}+n_{k_1},\cdots,k_j\notin d_{k_1k_2\cdots k_{j-1}}+\sum_{s=1}^{j-1}n_{k_s}\end{aligned} \tag{12}$$

$d_{k_1k_2\cdots k_j}$ is the number of links from the sender to node $k_1,k_2,\ldots,k_j$. So substituting the above equation into (9), one has

$$\begin{aligned}P(M=2)&=\sum_{k_1=1}^{N}p(1-p)^{N+d_k-1}+\frac{1}{2!}\sum_{k_1=1}^{N}\sum_{k_2\notin D_{k_1}}p^2(1-p)^{N+d_{k_1k_2}-2}\\&+\frac{1}{3!}\sum_{k_1=1}^{N}\sum_{k_2\notin D_{k_1}}\sum_{k_3\notin D_{k_1k_2}}p^3(1-p)^{N+d_{k_1k_2k_3}-3}+\cdots\end{aligned} \tag{13}$$

After expanding the above equation, one may obtain the following approximation.

$$P(M=2)=Np-[\binom{N}{2}+\sum_{k_1=1}^{N}(n_{k_1}+d_{k_1})]p^2+a_{23}p^3+\cdots \tag{14}$$

where

$$a_{23}=\frac{1}{3!}\sum_{k_1=1}^{N}\sum_{k_2\notin D_{k_1}}\sum_{k_3\notin D_{k_1k_2}}1+\sum_{k_1=1}^{N}\binom{N+d_k-1}{2}-\frac{1}{2!}\sum_{k_1=1}^{N}\sum_{k_2\notin D_{k_1}}(N+d_{k_1k_2}-2) \tag{15}$$

Similarly, one may use (14) to evaluate $P(M=2|k_1,k_2,\ldots,k_j)$. If one only considers 3rd order approximation, one has the following result.

$$P(M=3)=\sum_{k_1=1}^{N}(d_{k_1}+n_{k_1})p^2+a_{33}p^3+\cdots \tag{16}$$

where

$$\begin{aligned}a_{33}&=-\sum_{k_1=1}^{N}[(N-\frac{3}{2}+\frac{d_{k_1}-n_{k_1}}{2})(d_{k_1}+n_{k_1})+\sum_{k_2\in D_{k_1}}(d'_{k_2}+n'_{k_2})]\\&+\frac{1}{2!}\sum_{k_1=1}^{N}\sum_{k_2\notin D_{k_1}}(d_{k_1k_2}+n_{k_1}+n_{k_2})\end{aligned} \tag{17}$$

Using (16), one can obtain the 3rd approximation of P(M=4).

$$P(M=4)=a_{43}p^3+\cdots \tag{18}$$

where $a_{43}=\sum_{k_1=1}^{N}\sum_{k_2\in D_{k_1}}(d'_{k_2}+n'_{k_2})$ (19)

So far, we derive till 3rd order approximation for probability density of the number of transmissions and retransmissions. One may easily find the expected number of transmissions.

$$E[M]=\sum_{m=1}^{\infty}mP(M=m)\approx 1+Np+[xN-\binom{N}{2}]p^2+[\binom{N}{3}+y]p^3 \tag{20}$$

where $x=\frac{1}{N}\sum_{k=1}^{N}(n_k+d_k)$ (21)

$$y=-2\binom{N}{3}+2a_{23}+3a_{33}+4a_{43} \tag{22}$$

After some calculations, y may be simplified to the following formula (appendix A).

$$y=\frac{N(11x+7)}{24}+\frac{7}{12}\sum_{k_1=1}^{N}(d_{k_1}+n_{k_1})^2-\sum_{k_1=1}^{N}[\frac{1}{3}(d_{k_1})^2+\frac{17}{12}(n_{k_1})^2] \quad (23)$$

2.4 Discussions

From (20), E[M] depends on the number of links N, packet loss probability p and topology. For different topologies, x and y value are different. x and y reflect effects of the 2nd and 3rd approximation of topology, respectively.

From lemma 1 in appendix A, we have $\sum_{k=1}^{N}n_k=\sum_{k=1}^{N}(d_k-1)$, so x is written as follows.

$$x=\frac{1}{N}\sum_{k=1}^{N}(n_k+d_k)=\frac{2}{N}\sum_{k=1}^{N}n_k+1=\frac{2}{N}\sum_{k=1}^{N}d_k-1 \quad (24)$$

Defining g_t as the probability that a node is in level t, that is, $g_t=G_t/N$ where G_t is the total number of links for level t, x can be written as,

$$x=\frac{2}{N}\sum_{k=1}^{N}d_k-1=\frac{2}{N}\sum_{t=1}^{L}tG_t-1=2\sum_{t=1}^{L}tg_t-1=2E[t]-1 \quad (25)$$

where E[t] is the expected value of node levels over the whole network for a general topology.

x reaches the biggest value for linear and the smallest value for star topology. For linear topology, one has $n_k+d_k=N$. It is easy to get x and y value from (24) and (23): x=N and $y=N^2$. For star topology, one has d_k=1 and n_k=0, so x=1 and $y=N$ from (24) and (23). As further approximation, y is ignored because $y<<\binom{N}{3}$. This approximation (20) for star and linear topology coincides perfectly with exact solution till the 3rd result.

We can compare our analytical approximation with previous results of recursions in the following. As shown in Fig. 2 and Fig. 3, these are two examples of topologies: type A and type B. For type A, each node has the same number of children nodes, i.e, w children nodes. Thus we obtain the following x value for k-ary tree.

$$x=2\sum_{k=1}^{N}n_k+1=2[L-\frac{w+1}{2(w-1)}]+\frac{2Lw}{N(w-1)} \quad (26)$$

where L is depth of k-ary tree and w is the number of children nodes for each node.

For type B, only one node in each level has children links, thus, we obtain x value from (24).

$$x=\frac{2}{N}\sum_{k=1}^{N}d_k-1=\frac{2}{N}\sum_{t=1}^{L}tw_t-1 \quad (27)$$

where w_t is the total number of links for level t, N is the total number of links. For a special case of $w_1=w_2=...=w_L=w$, we have N=Lw and x=L from (27).

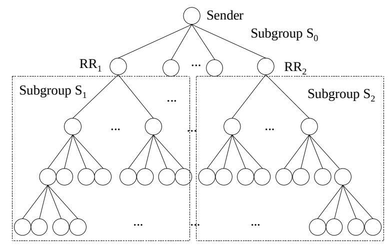

Fig. 2 Topology of type A - k-ary tree. w is the number of branches for each node.

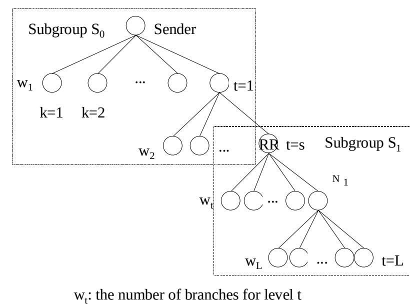

Fig. 3 Topology of type B. Only one node in each level has children nodes except the last level.

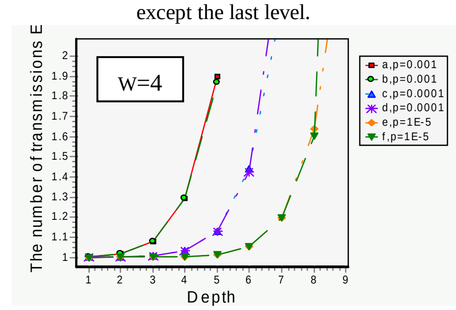

Fig. 4 Comparison of E[M] for type A. a,c,e: exact solution, b,d,f: approximate solution.

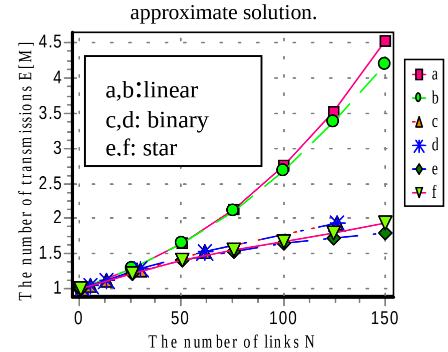

Fig. 5 Comparison of E[M] a,c,e: exact solution, b,d,f: approximate solution.

We compare approximate approach and exact approach for some topologies. Fig. 4, Fig. 5 and Fig. 6 give comparison of several topologies for the exact (i.e, the earlier recursion solution) and approximate solutions (20) for E[M]. For small E[M] value, (20) gives a very good approximation to the exact solution.

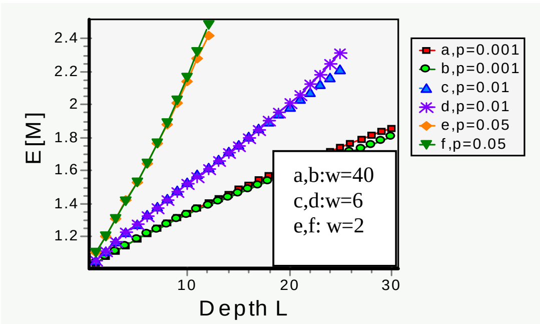

Fig. 6 Comparison of E[M] for type B. $w_1=w_2=w_3=...=w_L=w$. a,c,e: exact solution,

3. Optimal Placements

For tree-based multicast protocols, the receivers are organized hierarchically in a tree. Some special receivers or routers, for example, Designated Receivers (DR) in RMTP (reliable multicast transport protocol [1]), manage a group of receivers or a domain. In this paper, we will use repair routers (RR) to represent these special receivers or routers, that is, RRs have repair functions and handle retransmissions. In fact, the multicast group is partitioned into different subgroups according to these repair routers.
Placing r RRs that can send repairs to requesting nodes, one has r+1 subgroups. For each retransmission in one subgroup, the packet is always multicast to the whole node population of the subgroup by the parent RR. Most nodes will be affected by retransmissions. As a worst case estimate of bandwidth consumption, one can assume that all links in a subgroup will be affected by each retransmission. We define C_i as the total bandwidth consumed by one source multicast packet over all links in subgroup i whose retransmissions are handled by one RR. Thus, one may obtain the expected bandwidth of each subgroup.

$$E[C_i] \approx E[M_i]N_i. \tag{28}$$

where $E[C_i]$ and $E[M_i]$ are the expected bandwidth consumption and the expected number of transmissions and retransmissions for subgroup i respectively, and N_i is the number of links for subgroup i.

Bandwidth consumption of the whole multicast group is summation of bandwidth consumed by all subgroups. Therefore, for r RRs, one may find the total expected bandwidth consumption E[C] [6].

$$E[C]=\sum_{i=0}^{r} E[C_i]=\sum_{i=0}^{r} N_i E[M_i] \tag{29}$$

Suppose that $E[M_i]= f_i(N_i)$, i=0,1,...,r. If B is defined as the bandwidth consumed by a multicast packet per link, averaged over all links in a multicast group, thus one may obtain the following equation.

$$E[B]=\frac{E[C]}{N}=\frac{1}{N}\sum_{i=0}^{r} N_i f_i(N_i) \tag{30}$$

One may use Lagrange multipliers to obtain optimal condition, that is, we need to minimize the function subject to the following constraint.

$$\sum_{i=0}^{r} N_i - N = 0 \tag{31}$$

Introducing Lagrange multiplier λ, we form new objective function $g(N_0,N_1,..., N_r)$ to be minimized.

$$g(N_0,N_1,\cdots,N_r)=\frac{1}{N}\sum_{i=0}^{r} N_i f_i(N_i)+\lambda(\sum N_i - N) \tag{32}$$

We find its partial derivatives with respect to N_0, N_1,..., N_r and set them equal to zero.

$$\frac{\partial}{\partial N_i} g(N_0,N_1,\cdots,N_r)=\frac{1}{N}[\, f_i(N_i)+N_i\frac{\partial f_i(N_i)}{\partial N_i}]+\lambda=0, \qquad i=0,1,\cdots,r. \tag{33}$$

Thus we have the following optimal condition of r RRs.

$$f_0(N_0)+N_0\frac{\partial f_0(N_0)}{\partial N_0}=f_1(N_1)+N_1\frac{\partial f_1(N_1)}{\partial N_1}=\cdots=f_r(N_r)+N_r\frac{\partial f_r(N_r)}{\partial N_r} \tag{34}$$

where $f_i(N_i)$ is given by (20) for subgroup i.

3.1 Homogeneous Topology Structure Of Different Subgroups

If each subgroup has the same topology, then $E[M_0]$ and $E[M_1]$ should have the same functional dependence on the number of links. Many topologies fit into this case. In the case of linear topology, each subgroup always has the same topology no matter where RRs are placed. For topology of type A, all subgroups have similar topology. For topology of type B with $w_1=w_2=...=w_L$, they also have the same topologies when RRs are placed in the intermediate nodes. From analytical approximation (20), if each subgroup has the same x function of N in the case that y is neglected, they have the same f(N) function. From (20) and (25), as long as each level has the same number of links for each subgroup, these subgroups have the same function x and the same functional dependence on the number of links N. In the case of the same functional dependence with N, that is, $f_0(N_0)=f_1(N_1)=...=f_r(N_r)=f(N)$, or $x_0(N_0)=x_1(N_1)=...=x_r(N_r)$, the optimal conditions of RRs are obvious from **(34)**:

$$N_0=N_1=N_2=...=N_r \tag{35}$$

Therefore, RR should be placed optimally to partition the group into such subgroups having the same number of links. This applies to homogeneous topology.

3.2 Different Topology Structure For Different Subgroups

If each subgroup has a different topology, then $E[M_0]$ and $E[M_1]$ will have different functional dependence on the number of links, for example, for type B, if $w_1=w>1$, $w_2=...=w_L=1$, one subgroup is linear topology and another is like star topology when 1 RR is placed in t^{th} level. For the same number of links N, linear topology has much larger E[M] than star topology. For example, if two subgroups have very different functional dependence, that is, $f_0(N_0) \neq f_1(N_1)$, $N_0=N_1$ will not be optimal conditions for this case. In order to satisfy optimal condition **(34)**, linear-like subgroup should have small size while star-like subgroup should have larger size.

3.3 Results And Discussions

From the above analysis, placements of RRs depend on difference of topologies among subgroups. For the homogeneous topology structure, they have similar topologies after partitioning according to RR locations. Thus, optimal placements depend on only the number of links of each subgroup, that is, each subgroup should have the same size. We use type A to illustrate this case. When RRs are placed in network of type A, each subgroup has similar topology. Thus, RRs are always placed to have the same topology and the same number of links for each subgroup. We can see these results from Table 2. For example, levels 1,1,2 are the optimal placements for 3 RRs in the case of 3-ary tree because this is the best RR combination having the same topology and the same size for 4 subgroups. For pure linear topology, we also have similar results. No matter how we partition the linear topology, each subgroup always has the linear topology. Thus each subgroup should have the same number of links to get the best performance of multicast. This result coincides with results of linear topology in [2]. Optimal placements of RRs are trade-off between the sender and each RR. When each subgroup has the same number and the same topology, the whole group has the best performance.

Table 2. Optimal levels of RRs for type A

#RRs \ w	2	3	4	5
2 RRs	1,2	1,1	1,1	1,1
3 RRs	2,2,2	1,1,2	1,1,1	1,1,1

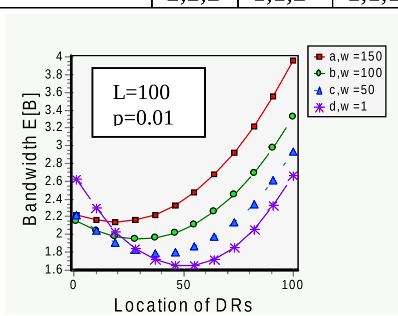

Fig. 7 Bandwidth E[B] for type B using 1 RR. w_1=w, w_2=...=w_L=1.Note w_1=1 is homogeneous topology structure

When each subgroup has very different topology, results of having the same size for each subgroup are not applicable. Fig. 7 give the optimal placements of RRs for topology of type B where w_1=w and w_2=...=w_L=1. Location of RR is level of RR. For example, optimal placement of curve a in Fig. 7 is in level 20, that means, N_0=w+20=170 and N_1=100-20=80. We have different number of links for each subgroup to have the optimal placements of RRs. When we have the same size, N_0=100 and N_1=100 of curve b in Fig. 7 i.e, RR is in level 1, multicast performance of type B is not optimal. The same number of links for each subgroup doesn't result in the best performance of multicast for very different topology. Subgroup S_1 is linear topology and subgroup S_0 is like star topology. Their topologies are very different. Optimal results are (found by numerical enumeration of (34) over all RR locations) that S_1 should have less number of links and S_0 should have more number of links. It may as well that linear topology has a larger number of retransmissions than star topology for the same number of links, thus linear topology should have less number of links than star topology so that the whole population reaches a trade-off between two subgroups.

4. Conclusions

The number of transmission and retransmissions is a very important parameter. In this work, we have derived a good analytical approximation for the number of transmission and retransmissions, which depends on the total number of links, loss probability and topology. x approximately reflects effects of topology on multicast performance. x reaches the biggest value for linear and the smallest value for star topology. We can use the number of transmissions to easily evaluate the bandwidth consumption.

Based on estimation of bandwidth consumption, effect of topology on optimal RR placements has been investigated for multicast in this work. For the same topology structure, the multicast group should be partitioned to have the same size of each subgroup so that the whole group has the best performance, the same can't be said for topologies where each subgroup has very different topology structures.

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APPENDIX A

Lemma 1: $$\sum_{k=1}^{N} n_k = \sum_{k=1}^{N}(d_k - 1) \quad (36)$$

Proof: Suppose that each node has one counter whose value is 0 at the beginning. Every time one adds the quantity d_k for node k to $\sum_{k=0}^{N} d_k$, then these counters whose nodes belong to the path from the sender to node k will increase by 1. Finally, $\sum_{k=0}^{N} d_k$ is summation of counters for all nodes. For a specific node k, its counter is determined by the number of downstream nodes n_k, i.e, its counter will be increased by n_k times. Thus $\sum_{k=0}^{N} d_k$ is summation of n_k for all nodes including the sender (node 0), i.e, $\sum_{k=0}^{N} d_k = \sum_{k=0}^{N} n_k$. Thus we have $\sum_{k=1}^{N} n_k = \sum_{k=1}^{N}(d_k - 1)$.

Lemma 2: $$\sum_{k_1=1}^{N} \sum_{k_2 \in d_{k_1}} n_{k_2} = \sum_{k_1=1}^{N}(n_{k_1} + 1)n_{k_1} \quad (37)$$

Proof: Suppose that each node has one counter whose value is 0 at the beginning. For each node from the sender to the specific node k_1, $\sum_{k_2 \in d_{k_1}} n_{k_2}$ will increase a value to its counter each time that is the number of links downstream. For a specific node k, its counter will be increased n_k+1 times determined by the number of downstream nodes including node k itself. The final value of counter for node k is $n_k(n_k+1)$. Thus $\sum_{k_1=1}^{N} \sum_{k_2 \in d_{k_1}} n_{k_2}$ is summation of all node counters, i.e, $\sum_{k_1=1}^{N} \sum_{k_2 \in d_{k_1}} n_{k_2} = \sum_{k_1=1}^{N}(n_{k_1} + 1)n_{k_1}$.

Lemma 3: $$\sum_{k_1=1}^{N} \sum_{k_2 \in n_{k_1}} n_{k_2} = \sum_{k_1=1}^{N}(d_{k_1} - 1)n_{k_1} \quad (38)$$

Proof: Suppose that each node has one counter whose value is 0 at the beginning. For each node under the specific node k_1, $\sum_{k_2 \in n_{k_1}} n_{k_2}$ will increase a value to its counter each time that is the number of links downstream. For a specific node k, its counter will be increased d_k-1 times determined by the number of links not including node k itself. The final value of counter for node k is $n_k(d_k-1)$. Thus $\sum_{k_1=1}^{N} \sum_{k_2 \in n_{k_1}} n_{k_2}$ is summation of all node counters, i.e, $\sum_{k_1=1}^{N} \sum_{k_2 \in n_{k_1}} n_{k_2} = \sum_{k_1=1}^{N}(d_{k_1} - 1)n_{k_1}$.

From the above lemmas, one has the following equations

$$\sum_{k_1=1}^{N} \sum_{k_2 \in n_{k_1}} d_{k_2} = \sum_{k_1=1}^{N}[(d_{k_1}+1)n_{k_1} + \sum_{k_2 \in n_{k_1}} n_{k_2}] = 2\sum_{k_1=1}^{N} d_{k_1} n_{k_1} \quad (39)$$

$$\sum_{k_1=1}^{N} \sum_{k_2 \in d_{k_1}} d_{k_2} = \frac{1}{2}\sum_{k_1=1}^{N}(d_{k_1}+1)d_{k_1} \quad (40)$$

$$\sum_{k_1=1}^{N} \sum_{k_2 \in D_{k_1}} d_{k_2} = \sum_{k_1=1}^{N}[\frac{1}{2}(d_{k_1}+1)d_{k_1} + 2d_{k_1}n_{k_1}] \quad (41)$$

$$\begin{aligned}\sum_{k_1=1}^{N} \sum_{k_2 \notin D_{k_1}} (n_{k_1} + n_{k_2}) &= \sum_{k_1=1}^{N} n_{k_1}(N - d_{k_1} - n_{k_1}) + \sum_{k_1=1}^{N}\sum_{k_2=1}^{N} n_{k_2} - \sum_{k_1=1}^{N}\sum_{k_2 \in D_{k_1}} n_{k_2} \\ &= N^2(x-1) - 2\sum_{k_1=1}^{N} n_{k_1}(d_{k_1} + n_{k_1})\end{aligned} \quad (42)$$

$$\frac{1}{2!}\sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}} 1 = \frac{1}{2!}\sum_{k_1=1}^{N}(\sum_{k_2=1}^{N} 1 - \sum_{k_2 \in D_{k_1}} 1)] = \frac{N(N-x)}{2} \quad (43)$$

$$\begin{aligned}\sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}} d_{k_1k_2} &\approx \sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}} (d_{k_1} + d_{k_2}) \\ &= \sum_{k_1=1}^{N}[\sum_{k_2=1}^{N}(d_{k_1} + d_{k_2}) - \sum_{k_2 \in D_{k_1}}(d_{k_1} + d_{k_2})] \\ &= N^2(x+1) - \frac{N(x+1)}{4} - \sum_{k_1=1}^{N}[\frac{3}{2}(d_{k_1})^2 + 3d_{k_1}n_{k_1})\end{aligned} \quad (44)$$

$$\begin{aligned}\sum_{k_1=1}^{N}\sum_{k_2 \in D_{k_1}} (d'_{k_2} + n'_{k_2}) &= \sum_{k_1=1}^{N}\sum_{k_2 \in d_{k_1}} (d'_{k_2} + n'_{k_2}) + \sum_{k_1=1}^{N}\sum_{k_2 \in n_{k_1}} (d'_{k_2} + n'_{k_2}) \\ &= \sum_{k_1=1}^{N} d_{k_1}(d_{k_1} + n_{k_1}) + \sum_{k_1=1}^{N}\sum_{k_2 \in n_{k_1}} (d_{k_2} + n_{k_2}) = \sum_{k_1=1}^{N}[(d_{k_1})^2 + 4d_{k_1}n_{k_1} - n_{k_1}]\end{aligned} \quad (45)$$

$$\begin{aligned}\frac{1}{3!}\sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}}\sum_{k_3 \notin D_{k_1}, D_{k_2}} 1 &= \frac{1}{3!}\sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}}(N - d_{k_1k_2} - n_{k_1} - n_{k_2}) \\ &= \frac{N^2(N-x)}{6} - \frac{1}{3!}\sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}}(d_{k_1k_2} + n_{k_1} + n_{k_2})\end{aligned} \quad (46)$$

$$\sum_{k_1=1}^{N}\binom{N+d_k-1}{2} = \frac{N(N-2)(N+x)}{2} + \frac{N(x+1)}{4} + \sum_{k_1=1}^{N}\frac{1}{2}(d_k)^2 \quad (47)$$

From (22), y is rewritten as the follows.

$$\begin{aligned}y = &-2\binom{N}{3} + 2[\frac{1}{3!}\sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}}\sum_{k_3 \notin D_{k_1k_2}} 1 + \sum_{k_1=1}^{N}\binom{N+d_k-1}{2} \\ &- \frac{1}{2!}\sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}}(N + d_{k_1k_2} - 2)] - 3\sum_{k_1=1}^{N}(N - \frac{3}{2} + \frac{d_{k_1} - n_{k_1}}{2})(d_{k_1} + n_{k_1}) \\ &+ \frac{3}{2!}\sum_{k_1=1}^{N}\sum_{k_2 \notin D_{k_1}}(d_{k_1k_2} + n_{k_1} + n_{k_2}) + \sum_{k_1=1}^{N}\sum_{k_2 \in D_{k_1}}(d'_{k_2} + n'_{k_2})\end{aligned} \quad (48)$$

Substituting (36) -- (47) into the above equation, we obtain y

$$y = \frac{N(11x+7)}{24} + \frac{7}{12}\sum_{k_1=1}^{N}(d_{k_1} + n_{k_1})^2 - \sum_{k_1=1}^{N}[\frac{1}{3}(d_{k_1})^2 + \frac{17}{12}(n_{k_1})^2] \quad (49)$$